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Energy–Efficiency Fairness of Interference Multi-Relay Networks for Multi-User Communications

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***Abstract*—We investigate a single-antenna multi-user multi-relay *interference* network, where multiple source nodes simultaneously communicate with their respective destination nodes via half-duplex decode-and-forward relays. To ensure fairness among users, we consider a power allocation strategy to maximize the worst-case energy efficiency (EE) of all users for a fixed relay assignment. The resulting optimization problem turns out to be non-convex. Different from those in the literature, our method here is an iterative algorithm where two geometric programs (GPs) are solved in each iteration, one producing an upper-bound to the solution of the original problem and the other providing a feasible lower-bound. Moreover, the upper-bound GP approaches the original problem asymptotically. Our algorithm also works for the problem arising in the non-interfering (orthogonal) transmission, which was previously solved as a fractional program. Numerical results reveal that non-orthogonal transmission outperforms orthogonal transmission in terms of the worst-case EE at low and medium signal-to-noise ratios.**

***Index Terms*—Energy efficiency, interference channels, max-min fairness, optimal solution methods, user fairness.**

I. Introduction

Energy efficiency (EE) is considered a key performance metric in the design of 5G wireless networks as it is a function of data rate, power consumption and frequency/time resource usage [1], [2]. On the other hand, the use of relaying with proper resource allocation improves the outage, coverage and connectivity with quality-of-service guarantees in multi-user networks [3], where orthogonal transmission is often employed to mitigate multi-user interference [4]. Due to the bandwidth limitation, orthogonal transmission via relays might be inefficient in connecting multiple end-to-end devices, e.g., Internet-of-Things (IoT). Thus, resource allocation for energy efficient transmission over *non-orthogonal* channels in a relay network is the focus of this paper. The non-orthogonal transmission here forms a relay-assisted *interference channel*, which is fundamentally different from non-orthogonal multiple access (NOMA) [5]. In wireless networks, it is not uncommon that transmissions of users with unfavorable channel conditions suffer from at least temporary outages in order to boost some system-wide objectives. Thus, to ensure *fairness* among users, for the multi-user multi-relay network considered here, the worst-case EE among all users is maximized.

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A. Related Work

For orthogonal transmission in relay networks, the EE has been widely considered in different system models, e.g., single-user multi-relay networks, multi-user single-relay networks, and multi-user multi-relay networks (see [6]–[8] and references therein). However, for non-orthogonal transmission or interference channels, while the EE has been extensively studied in point-to-point networks or downlink multiple-input multiple-output (MIMO) multi-cell systems (see, e.g., [9] and references therein), very limited work is available on the EE in relay-assisted networks. Furthermore, in all the available work on multi-user relay networks with non-orthogonal transmissions [5], [10]–[13], the objective was to maximize *the global EE*. More specifically, single-relay networks are considered for one-way decode-and-forward (DF) relaying in [10], two-way amplify-and-forward (AF) relaying in [11], wireless-powered massive MIMO aided multi-way AF relaying in [12] and NOMA full-duplex AF relaying in [5]. In contrast, a multi-relay network is considered for energy harvesting in [13].

B. Our Contribution

Our contribution is two-fold. 1) We consider non-orthogonal transmissions in a multi-user multi-relay network, where joint power allocation at source transmitters and DF relays is performed to maximize *the worst-case EE or the EE fairness* among all users. The advantage of the proposed non-orthogonal transmissions over the orthogonal transmissions in [7] is explored with extensive numerical simulations. 2) The problem under consideration *does not* admit a straightforward convex formulation. The method proposed in [7], which is based on fractional programming, does not apply directly when non-orthogonal transmissions are employed. Thus we propose an iterative algorithm, which computes a sequence of upper bounds that *converges to the optimal value of the problem*. The main computations of the method are based on geometric programs (GPs), which are solved in polynomial time [14]. In addition, the proposed method terminates by returning a feasible point with an optimality certificate. Simulation results suggest that a feasible point of high accuracy is obtained in a *few* iterations of the algorithm.

II. System Model and Problem Formulation

A. Network Model

The direct channel between a particular source and destination pair, termed as the SD pair, becomes weak due to obstacles and/or deep fading. An intermediate DF relay node can then be employed to maintain the link. Thus, our system model is a dual-hop wireless relay network as shown in Fig. 1. Here the K source nodes, $S_1, \cdots, S_K$, indirectly send information to their corresponding destination nodes, $D_1, \cdots, D_K$, via a set of K DF relays, $R_1, \cdots, R_K$. The

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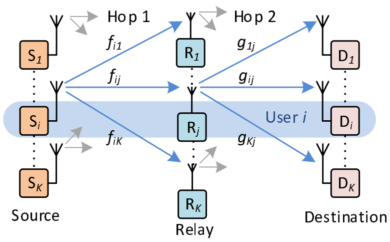

Fig. 1: A multi-user multi-relay network model.

i-th SD pair, i.e., S_i–D_i, is simply referred to as user i, $i = 1, \ldots, K$. Each node is equipped with a single antenna. We assume that the relay associated with user i is pre-assigned[1] and denoted as R_i, $i = 1, 2, \ldots, K$, without loss of generality. The link S_i–R_i–D_i is termed as the i-th link.

Denote power budgets for the i-th source S_i and the i-th relay R_i as p_i and q_i, $i = 1, \ldots, K$, respectively. During the first hop, the channel coefficient between the source S_i and relay R_j is denoted by f_{ij}, $i, j = 1, \ldots, K$, which are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables with variance σ_f^2, denoted by $\mathcal{CN}(0, \sigma_f^2)$. Similarly, in the second hop, the channel coefficient between the relay R_j and the destination D_i is denoted by g_{ij}, $i, j = 1, \ldots, K$, which are i.i.d. $\mathcal{CN}(0, \sigma_g^2)$. Note that path losses or shadowing can be subsumed into channel coefficients. The additive white Gaussian noise (AWGN) samples at different relay nodes and destination nodes are also mutually independent, and are i.i.d. $\mathcal{CN}(0, \sigma_r^2)$ and $\mathcal{CN}(0, \sigma_d^2)$, respectively.

B. Analytical Model

Two methods of transmission are adopted.

1) Non-Orthogonal Transmission (NOT): With this transmission, user pairs are transmitting simultaneously, thereby forming an interference channel. The effective end-to-end received signal-to-interference-plus-noise (power) ratio (SINR) of the i-th link is given by

$$\gamma_i^{\mathtt{NOT}}(\mathbf{p}, \mathbf{q}) = \min\left(\gamma_{1,i}^{\mathtt{NOT}}(\mathbf{p}), \gamma_{2,i}^{\mathtt{NOT}}(\mathbf{q})\right), \tag{1}$$

where $\mathbf{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$, $\mathbf{q} = [q_1, \ldots, q_K]^{\mathrm{T}}$, $\gamma_{1,i}^{\mathtt{NOT}}$ and $\gamma_{2,i}^{\mathtt{NOT}}$ are the SINRs of the first and second hops, respectively, i.e.,

$$\gamma_{1,i}^{\mathtt{NOT}}(\mathbf{p}) = \frac{p_i |f_{ii}|^2}{\sum_{\substack{j=1\\ j\neq i}}^{K} p_j |f_{ji}|^2 + \sigma_r^2}, \quad \gamma_{2,i}^{\mathtt{NOT}}(\mathbf{q}) = \frac{q_i |g_{ii}|^2}{\sum_{\substack{j=1\\ j\neq i}}^{K} q_j |g_{ij}|^2 + \sigma_d^2},$$

$i = 1, \ldots, K$. The EE for user i is given by

$$\eta_i^{\mathtt{NOT}}(\mathbf{p}, \mathbf{q}) = \frac{1}{2} \frac{\ln(1 + \gamma_i^{\mathtt{NOT}}(\mathbf{p}, \mathbf{q}))}{(p_i + q_i + c)}, \tag{2}$$

with its unit in nats/channel use/Joule, where the scaling factor $1/2$ is due to the dual-hop half-duplex relaying, and $c > 0$ is the static power dissipation to operate the communication of user i, which is assumed the same for all the links.

[1]Note that in general there may be N relays, $N \geq K$, available to help maintain the K SD links. Therefore, a proper relay assignment can also be implemented to achieve a particular objective, e.g., [15].

2) Orthogonal Transmission (OT): In this transmission, the channels of the user pairs are orthogonal by means of time-division multiple access (TDMA) or frequency-division multiple access. Orthogonal transmission is more popular, since ideally there is no interference. The effective end-to-end received signal-to-noise ratio (SNR) of the i-th link is given by

$$\gamma_i^{\mathtt{OT}}(p_i, q_i) = \min\left(\gamma_{1,i}^{\mathtt{OT}}(p_i), \gamma_{2,i}^{\mathtt{OT}}(q_i)\right), \tag{3}$$

where the SNRs of the first and second hops are given by

$$\gamma_{1,i}^{\mathtt{OT}}(p_i) = \frac{p_i |f_{ii}|^2}{\sigma_r^2} \quad \text{and} \quad \gamma_{2,i}^{\mathtt{OT}}(q_i) = \frac{q_i |g_{ii}|^2}{\sigma_d^2}, \tag{4}$$

respectively. The EE for user i is given by

$$\eta_i^{\mathtt{OT}}(p_i, q_i) = \frac{1}{2K} \frac{\ln(1 + \gamma_i^{\mathtt{OT}}(p_i, q_i))}{(p_i + q_i + c)}, \tag{5}$$

where the factor $1/2$ is due to half-duplex relaying as in NOT. The factor $1/K$ comes due to the inherent orthogonal resource sharing in OT, which can be a major source of inefficiency. Moreover, TDMA-based OT relies on stringent timing to avoid multi-user interference, whereas NOT does not emphasize this requirement.

C. Problem Formulation

We consider the scenario where both *energy efficiency* and *user fairness* are of crucial importance. In this context, a standard objective function is the *worst-case* (lowest) EE among all users with a given power budget for $\mathbf{p}$ and $\mathbf{q}$. Thus, the problem here is to maximize the worst-case EE among all users, and is formulated as

$$\begin{aligned} \text{maximize} \quad & \min_{i \in \{1,\ldots,K\}} \quad \eta_i^{\mathtt{NOT}}(\mathbf{p}, \mathbf{q}) \\ \text{subject to} \quad & \mathbf{0} \leq \mathbf{p} \leq \mathbf{1} P_{\mathtt{max}} \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{1} Q_{\mathtt{max}}, \end{aligned} \tag{6}$$

where the decision variables are $\mathbf{p}$ and $\mathbf{q}$. Here $\mathbf{1}$ and $\mathbf{0}$ denote the vectors with all-one and all-zero entries, respectively. The optimization problem for the case of orthogonal transmission is defined similarly by replacing $\eta_i^{\mathtt{NOT}}(\mathbf{p}, \mathbf{q})$ with $\eta_i^{\mathtt{OT}}(p_i, q_i)$.

Our assumptions of the channel state information (CSI) are similar to that of [7]. More specifically, at the initial stage of the communication, a central node is assumed to have access to global CSI via coordination, in order to solve problem (6) and obtain the optimal $\mathbf{p}$ and $\mathbf{q}$. The computed powers are then fed back to the respective transmit nodes, i.e., sources in the first hop and relays in the second hop. In the latter part of the communication, only the individual (local) receiver-side CSI is required at respective receivers to perform the decoding, i.e., the individual source-relay CSI at each relay node in the first hop and the individual relay-destination CSI at each destination node in the second hop. Clearly, if slow fading or block fading is assumed, the central node *does not need to frequently solve* problem (6), and all the nodes can perform transmission and/or reception independently without coordination during most of the channel coherence time.

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The equivalent epigraph form of problem (6) is given by

$$\begin{array}{ll} \text{maximize} & t \\ \text{subject to} & \mathbf{0} \le \mathbf{p} \le \mathbf{1} P_{\texttt{max}} \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{1} Q_{\texttt{max}} \\ & \gamma_{1,i}^{\texttt{NOT}}(\mathbf{p}) \ge e^{t(p_i+q_i+c)}-1,\ i \in \mathcal{K} \\ & \gamma_{2,i}^{\texttt{NOT}}(\mathbf{q}) \ge e^{t(p_i+q_i+c)}-1,\ i \in \mathcal{K}, \end{array} \tag{7}$$

where $\mathcal{K} = \{1, \ldots, K\}$ and the optimization variables are t, $\mathbf{p}$, and $\mathbf{q}$. It is not difficult to see that problem (7) is non-convex (e.g., see the last two constraints). Moreover, the problem does not admit a straightforward equivalent convex reformulation with standard tricks such as change of variables or objective/constraint function transformations, among others. As a result, solving problem (7) is challenging. Nevertheless, in the sequel, we derive a method that can efficiently solve problem (7) with a guaranteed optimality.

III. Solution for the Optimization Problem

The key idea of the proposed method is to generate a sequence of asymptotically tight *upper bounds* for the optimal value $t^\star$ of problem (7) in an iterative manner. In each iteration, an upper bound for $t^\star$ is computed from solving a *relaxed* problem, derived by *loosening* some constraints of the original problem (7). As will be shown later, the algorithm terminates in a finite number of iterations.

In addition, in each iteration, a lower bound for $t^\star$ is computed by solving a problem derived by *tightening* some constraints of the original problem (7). Solving the lower-bound problem provides a feasible point to (7). Furthermore, the difference between $t^\star$ and the optimal objective value of the lower-bound problem is at most the difference between the corresponding upper-bound and lower-bound values of $t^\star$.

Below we elaborate on the proposed method. While our discussion focuses on the problem of non-orthogonal transmission, one may find that the proposed method naturally works for the problem in orthogonal transmission considered in [7].

A. Computation of the Upper Bound $t_{\text{UB}}^{(l)}$ for $t^\star$

Consider the related problem

$$\begin{array}{ll} \text{maximize} & t \\ \text{subject to} & \mathbf{0} \le \mathbf{p} \le \mathbf{1} P_{\texttt{max}} \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{1} Q_{\texttt{max}} \\ & \gamma_{1,i}^{\texttt{NOT}}(\mathbf{p}) \ge \displaystyle\sum_{n=1}^{l} \frac{[t(p_i+q_i+c)]^n}{n!},\ i \in \mathcal{K} \\ & \gamma_{2,i}^{\texttt{NOT}}(\mathbf{q}) \ge \displaystyle\sum_{n=1}^{l} \frac{[t(p_i+q_i+c)]^n}{n!},\ i \in \mathcal{K}, \end{array} \tag{8}$$

with the variables t, $\mathbf{p}$, and $\mathbf{q}$, where the last two inequality constraints of problem (7) have been *relaxed* by using the fact that, for all $x \ge 0$ and $l \in \{1, 2, \ldots\}$, $e^x - 1 \ge \sum_{n=1}^{l} x^n/n!$.

Let $\mathcal{F}$ be the feasible set of problem (7). Moreover, let $\mathcal{F}_{\text{UB}}^{(l)}$ denote the feasible set of problem (8), where the superscript l on $\mathcal{F}$ denotes the number of terms in the summations on the right-hand side of the last two inequalities of problem (8). It is straightforward to verify that $\mathcal{F} \subset \mathcal{F}_{\text{UB}}^{(l)}$. As a result, we always have $t^\star \le t_{\text{UB}}^{(l)}$, where $t_{\text{UB}}^{(l)}$ is the optimal value of problem (8).

Interestingly, problem (8) is a geometric program (GP) [16, § 4.5]. This follows immediately since all the inequality constraints of problem (8) can readily be rearranged into the form of posynomial inequality constraints [16, § 4.5.2]. Therefore, problem (8) can be efficiently solved by using classic interior-point polynomial algorithms [14, § 6.3.1].

B. Computation of the Lower Bound $t_{\text{LB}}^{(l)}$ for $t^\star$

Recall that the lower bound $t_{\text{LB}}^{(l)}$ is computed by solving a problem whose constraints are tighter than those of the original problem (7). To this end, we first invoke a simple but useful inequality. Given $\hat{r}$, let $\hat{m} = \left(e^{\hat{r}} - 1\right)/\hat{r}$. Then

$$e^x - 1 \le \hat{m}x, \tag{9}$$

for all $0 < x \le \hat{r}$. Next, we formulate a problem related to (7), with the last two constraints of problem (7) replaced based on (9). This idea leads to the following GP:

$$\begin{array}{ll} \text{maximize} & t \\ \text{subject to} & \mathbf{0} \le \mathbf{p} \le \mathbf{1} P_{\texttt{max}} \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{1} Q_{\texttt{max}} \\ & \gamma_{1,i}^{\texttt{NOT}}(\mathbf{p}) \ge \hat{m}_i t(p_i+q_i+c),\ i \in \mathcal{K} \\ & \gamma_{2,i}^{\texttt{NOT}}(\mathbf{q}) \ge \hat{m}_i t(p_i+q_i+c),\ i \in \mathcal{K} \\ & t(p_i+q_i+c) \le \hat{r}_i,\ i \in \mathcal{K}, \end{array} \tag{10}$$

where the variables are t, $\mathbf{p}$, and $\mathbf{q}$. Here we have introduced new problem parameters $\hat{\mathbf{m}} = [\hat{m}_1, \ldots, \hat{m}_K]^{\text{T}}$ and $\hat{\mathbf{r}} = [\hat{r}_1, \ldots, \hat{r}_K]^{\text{T}}$, based on the spirit of (9). Note that

$$\hat{m}_i = \left(\exp(\hat{r}_i) - 1\right)/\hat{r}_i, \quad i \in \mathcal{K}. \tag{11}$$

According to (9), the value of $\hat{r}_i$ can be an arbitrary positive value, $\forall i \in \mathcal{K}$. In the proposed algorithm, we set $\hat{r}_i = t_{\text{UB}}^{(l)}\big(p_{i,\text{UB}}^{(l)} + q_{i,\text{UB}}^{(l)} + c\big)$, where $t_{\text{UB}}^{(l)}$, $(p_{i,\text{UB}}^{(l)}, q_{i,\text{UB}}^{(l)})_{i\in\mathcal{K}}$ are the solution of problem (8). By doing so, problem (10) inherits the superscript l corresponding to the l terms on the right-hand side of the last two inequality constraints of problem (8).

Since the constraints of problem (10) are tighter than those of problem (7) [*cf.* (9)], the feasible set $\mathcal{F}_{\text{LB}}^{(l)}$ of problem (10) and $\mathcal{F}$ are such that $\mathcal{F}_{\text{LB}}^{(l)} \subset \mathcal{F}$. Therefore, the optimal value $t_{\text{LB}}^{(l)}$ of problem (10) is always a lower bound for $t^\star$, i.e., $t_{\text{LB}}^{(l)} \le t^\star$. More importantly, the solution $t_{\text{LB}}^{(l)}$, $\big(p_{i,\text{LB}}^{(l)}, q_{i,\text{LB}}^{(l)}\big)_{i\in\mathcal{K}}$ of problem (10) is *feasible* for the original problem (7).

C. The Proposed Algorithm and Its Convergence

The proposed algorithm is summarized in Algorithm 1.

Recall that the upper limit of the summations of problem (8) is the iteration index l of Algorithm 1. Moreover, the larger the value of l, the higher the upper limit of the summations aforementioned. Steps 3 to 6 are simply the iterative computations of the upper-bound and lower-bound for the optimal value $t^\star$ of problem (7).

When $\left|t_{\text{UB}}^{(l-1)} - t_{\text{UB}}^{(l)}\right|$ is within the specified tolerance given by ϵ [*cf.* step 6], the algorithm terminates by returning the *best feasible* point [*cf.* step 7] computed so far. Let us define the accuracy of the feasible point by $\left|t^\star - t_{\text{LB}}^{(l^\star)}\right|$, which is clearly upper-bounded by $\left|t_{\text{UB}}^{(l^\star)} - t_{\text{LB}}^{(l^\star)}\right|$. Below we establish the convergence results of the algorithm.

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Algorithm 1 : Power Allocation for Fair Energy-Efficiency (PA-FEE)

1: *Initialization:* Set the iteration index $l = 0$ and the tolerance $\epsilon > 0$. l also denotes the number of terms in the summations of problem (8). $t_{\text{UB}}^{(0)} = 0$.
2: **repeat**
3: Set $l := l + 1$.
4: *Upper Bound:* Solve problem (8) to yield the solution $(t_{\text{UB}}^{(l)}, \mathbf{p}_{\text{UB}}^{(l)}, \mathbf{q}_{\text{UB}}^{(l)})$.
5: *Lower Bound:* Solve problem (10) with problem parameters (11) and $\hat{r}_i = t_{\text{UB}}^{(l)}\big(p_{i,\text{UB}}^{(l)} + q_{i,\text{UB}}^{(l)} + c\big), \forall i \in \mathcal{K}$ to yield solution $\big(t_{\text{LB}}^{(l)}, \mathbf{p}_{\text{LB}}^{(l)}, \mathbf{q}_{\text{LB}}^{(l)}\big)$. Set $l^\star = \arg\max_{n \in \{1,\ldots,l\}} t_{\text{LB}}^{(n)}$.
6: **until** $\big|t_{\text{UB}}^{(l-1)} - t_{\text{UB}}^{(l)}\big| < \epsilon$
7: *Output:* Return $t_{\text{LB}}^{(l^\star)}$, $\big(p_{i,\text{LB}}^{(l^\star)}, q_{i,\text{LB}}^{(l^\star)}\big)_{i \in \mathcal{K}}$ as the solution.

Lemma 1. *Consider the indexed collection of sets $\{\mathcal{F}_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$. The intersection of the collection is simply $\mathcal{F}$, i.e., $\mathcal{F} = \bigcap_{l \in \mathbb{N}} \mathcal{F}_{\text{UB}}^{(l)}$.*

Proof: $\forall x \geq 0$, $\lim_{l \to \infty} \sum_{n=1}^{l} (x^n/n!) = e^x - 1$. Furthermore, for $l \leq l'$ and $x \geq 0$, $\sum_{n=1}^{l} (x^n/n!) \leq \sum_{n=1}^{l'} (x^n/n!) \leq e^x - 1$, which means that $l \leq l' \implies \mathcal{F}_{\text{UB}}^{(l)} \supset \mathcal{F}_{\text{UB}}^{(l')} \supset \mathcal{F}$. ■

Corollary 1. *Consider the sequence $\{t_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$, where $t_{\text{UB}}^{(l)}$ is the first component of the solution $\big(t_{\text{UB}}^{(l)}, \mathbf{p}_{\text{UB}}^{(l)}, \mathbf{q}_{\text{UB}}^{(l)}\big)$ of problem (8) [cf. step 4 of Algorithm 1]. Then, $\lim_{l \to \infty} t_{\text{UB}}^{(l)} = t^\star$, where $t^\star$ is the optimal value of problem (7).*

Proof: Note that the sets $\mathcal{F}$ and $\mathcal{F}_{\text{UB}}^{(l)}$, $l \in \mathbb{N}$ in $\mathbb{R}^{2K+1}$ are *compact* and this property is implicitly applied in the rest of the proof. From the proof of Lemma 1, the collection of sets $\{\mathcal{F}_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$ constitutes a *monotone* sequence in the sense that $\mathcal{F}_{\text{UB}}^{(l)} \supset \mathcal{F}_{\text{UB}}^{(l+1)} \supset \cdots$. Moreover, $\mathcal{F}_{\text{UB}}^{(l)} \supset \mathcal{F}$ for all $l \in \mathbb{N}$. Thus, the sequence $\{t_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$ is monotonic and bounded below, i.e., $t_{\text{UB}}^{(l)} \geq t_{\text{UB}}^{(l+1)} \geq \cdots$ and $t_{\text{UB}}^{(l)} \geq t^\star$ for all $l \in \mathbb{N}$. Then it follows from [17, Theorem 3.14] that the sequence converges. Let $\bar{t}$ be the limit of the sequence, i.e., $\bar{t} = \lim_{l \to \infty} t_{\text{UB}}^{(l)}$. It remains to show that $\bar{t} = t^\star$. Clearly, $\bar{t} < t^\star$ is impossible since $t_{\text{UB}}^{(l)} \geq t^\star$ for all $l \in \mathbb{N}$. Suppose that $\bar{t} > t^\star$. Then

$$\limsup_{l \to \infty} \mathcal{F}_{\text{UB}}^{(l)} \text{ is a } \textit{proper superset} \text{ of } \mathcal{F}, \tag{12}$$

where the $\limsup$ operator is used to define the outer limit of the set sequence $\{\mathcal{F}_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$ (*cf.* [18, 4.1 Definition]). However,

$$\lim_{l \to \infty} \mathcal{F}_{\text{UB}}^{(l)} \overset{\text{(a)}}{=} \bigcap_{l \in \mathbb{N}} \mathcal{F}_{\text{UB}}^{(l)} \overset{\text{(b)}}{=} \limsup_{l \to \infty} \mathcal{F}_{\text{UB}}^{(l)} \overset{\text{(c)}}{=} \mathcal{F}, \tag{13}$$

where in the above (a) follows from the results pertaining to monotone sequences of set (*cf.* [18, p. 111]), (b) follows immediately from [18, 4.1 Definition], and (c) follows by Lemma 1. Thus, (12) and (13) stand in contradiction. ■

Proposition 1. *The Algorithm 1 (PA-FEE) terminates in a finite number of iterations.*

Proof: The sequence $\{t_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$ is convergent, *cf.* Corollary 1. Thus, the sequence $\{t_{\text{UB}}^{(l)}\}_{l \in \mathbb{N}}$ is a Cauchy sequence, *cf.* [17, Theorem 3.11-(a)], which ensures the termination of PA-FEE at step 6 of the algorithm. ■

IV. Numerical Results and Discussions

In all the simulations, we set $P_{\max} = Q_{\max}$ and $\sigma_f^2 = \sigma_g^2 = 1$. We assume $\sigma_r^2 = \sigma_d^2 = \sigma^2$, and define the signal-to-noise-ratio as $SNR = P_{\max}/\sigma^2$. We consider two static power dissipation models: i) $c = 0.01 P_{\max}$ and ii) $c = 0.01$ (W). The upper- and lower-bounds of the worst-case EE for non-orthogonal transmission are obtained from solving (8) and (10), respectively.

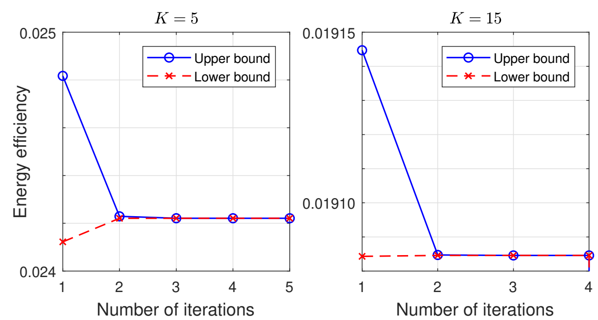
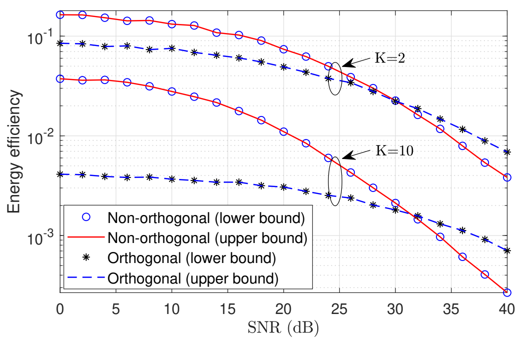


Fig. 2: (Worst-case) EE vs. number of iterations ($K = 5, 15$)

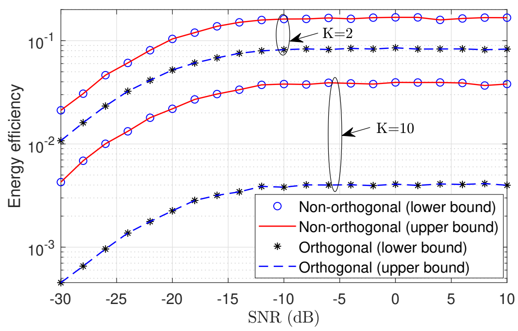
For a single channel (fading) realization, Fig. 2 plots the (worst-case) EE values of the upper- and lower-bounds versus the number of iterations when $SNR = 10\,\text{dB}$, $c = 0.01 P_{\max}$ and $\epsilon = 10^{-8}$. It can be seen that 5 and 4 iterations are required for $K = 5$ and 15, respectively. When $\epsilon = 10^{-6}$, however, 4 and 3 iterations are required for $K = 5$ and 15, respectively. Results show that PA-FEE yields a feasible point with high accuracy within only a few iterations, for all cases we have considered given that $\epsilon \geq 10^{-8}$. Since PA-FEE has fast convergence, it suits for practical implementations.

Next, for accuracy within 10^{-8} of the optimum, we obtain the average worst-case EE over 2000 channel realizations in Figs. 3 and 4. For both non-orthogonal and orthogonal transmissions with $K = 2$ and $K = 10$, Figs. 3a and 3b show the average worst-case EE vs. SNR for $c = 0.01 P_{\max}$ and $c = 0.01$ W, respectively. Several observations are in order. i) Both upper- and lower-bounds tightly match in all cases. ii) For $c = 0.01 P_{\max}$, the worst-case EE decreases when SNR increases and the decrease is larger for non-orthogonal transmission. For $c = 0.01$ W, the worst-case EE increases with SNR and reaches a plateau. iii) For $c = 0.01 P_{\max}$, while the non-orthogonal transmission outperforms the orthogonal transmission at low or medium SNRs, the opposite is observed at high SNRs. iv) For $c = 0.01$ W, the non-orthogonal transmission always outperforms the orthogonal transmission.

Figs. 4a and 4b show the average worst-case EE vs. the number of users (K) for $c = 0.01 P_{\max}$ and $c = 0.01$ W, respectively. For both methods of transmission, the worst-case EE decreases when K increases. For non-orthogonal transmissions, the decrease in the worst-case EE is caused by the increased interference. For orthogonal transmissions, the decrease comes from the inherent inefficiency of the transmission, *cf.* the ratio $1/K$ in (5). For $c = 0.01 P_{\max}$, at $SNR =$

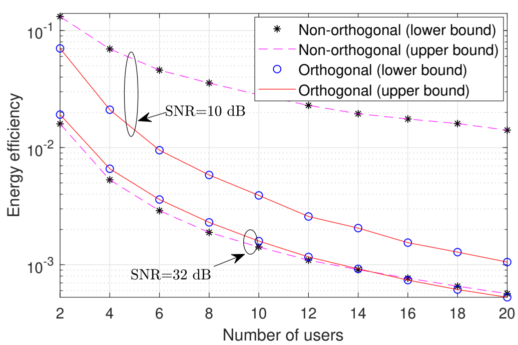


(a) $c = 0.01P_{\max}$.

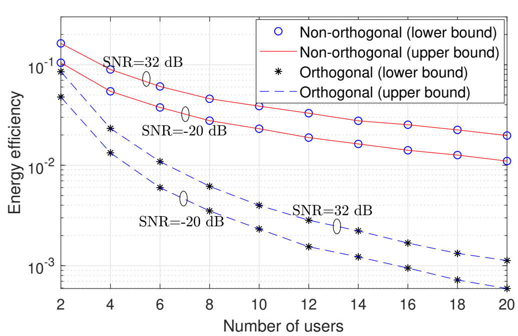


(b) $c = 0.01$ W.

Fig. 3: Average (worst-case) EE vs. SNR ($K = 2, 10$).



(a) $c = 0.01P_{\max}$ ($SNR = 10, 32$ dB).



(b) $c = 0.01$ W ($SNR = -20, 32$ dB).

Fig. 4: Average (worst-case) EE vs. number of users.

10 dB, when K increases from 2 to 20, the worst-case EE reduces by 89% for non-orthogonal transmissions and by 96% for orthogonal transmissions. However, at $SNR = 32$ dB, for $K < 14$, the orthogonal transmission has a better performance than the non-orthogonal transmission. Interestingly, for fixed $c = 0.01$ W, the non-orthogonal transmission outperforms the orthogonal transmission in the entire simulated SNR region.

V. Conclusion

The worst-case energy efficiency (EE) among all the links designated by source-relay-destination triads was maximized with respect to the transmit powers at sources and relays in a multi-user multi-relay interference network. A novel iterative algorithm based on geometric programming was proposed to find a feasible point with guaranteed optimality for the resulting non-convex problem. Convergence of the algorithm was analytically established. Simulations show that convergence to within 10^{-8} of the optimum is achieved within a few iterations. In terms of the worst-case EE, the non-orthogonal transmission performs better at low to medium SNR values than the orthogonal transmission.

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