

Auction-Based Resource Allocation in MillimeterWave Wireless Access Networks

George Athanasiou, *Member, IEEE*, Pradeep Chaturanga Weeraddana, *Member, IEEE*,
and Carlo Fischione, *Member, IEEE*

Abstract---The resource allocation problem of optimal assignment of the stations to the available access points in 60 GHz millimeterWave wireless access networks is investigated. The problem is posed as a multi-assignment optimization problem. The proposed solution method converts the initial problem to a minimum cost flow problem and allows to design an efficient algorithm by a combination of auction algorithms. The solution algorithm exploits the network optimization structure of the problem, and thus is much more powerful than computationally intensive general-purpose solvers. Theoretical and numerical results evince numerous properties, such as optimality, convergence, and scalability in comparison to existing approaches.

Index Terms---Auction-based resource allocation.

I. INTRODUCTION

MILLIMETERWAVE (mmW) wireless networks in the 60 GHz unlicensed band are considered one of the key technologies for enabling multi-gigabit wireless access (transmission rates up to 7 Gbps) and provisioning of QoS-sensitive applications [1]. More than 5 GHz of continuous bandwidth is available in many countries worldwide, which makes 60 GHz systems particularly attractive for gigabit wireless applications such as gigabyte file transfer, wireless docking station, wireless gigabit ethernet, and uncompressed high definition video transmission. Moreover, scenarios such as dense small-cells and mobile data offloading [2], which are strongly motivated by the increased end-user connectivity requirements and mobile traffic, can be accommodated with the use of 60 GHz radio access technology.

Resource allocation for wireless local area networks has been the focus of intense research. Several studies have analyzed the performance of the basic station (STA) association policy that IEEE 802.11 standard defines, based on the received signal strength indicator (RSSI). These studies have showed that this basic association policy can lead to inefficient use of the network resources [3]. Therefore, there has been increasing interest in designing better STA association policies [4]--[6]. Previous approaches are hard to apply in 60 GHz wireless access networks due to the special characteristics of the 60 GHz channel, and the differences with the rest wireless access technologies [7]--[10] (namely, severe channel attenuations, high path loss, directionality, and blockage), novel mechanisms must be designed to provide optimal resource allocation. Our previous approach [11] was the first to study the STA association in 60 GHz wireless access networks. However, the focus was on the network performance (load balancing) and not on optimizing the benefit of the STAs.

Manuscript received August 5, 2013. The associate editor coordinating the review of this letter and approving it for publication was D. W. K. Ng.

The authors are with the Electrical Engineering School, ACCESS Linnaeus Center, KTH Royal Institute of Technology, Stockholm, Sweden (e-mail: {georgioa, chatw, carlofi}@kth.se).

This paper is supported by the EU projects HydroBioNets and Hycon2. Digital Object Identifier 10.1109/LCOMM.2013.100713.131781

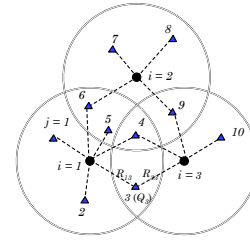


Fig. 1. Example mmW wireless access network.

This paper poses the STA association optimization problem, where the objective is to *maximize the total weighted throughput* that the STAs achieve (the weights are affected by the STAs requests). Such a problem is more challenging in 60 GHz band than in traditional wireless networks since the wireless channel is unstable and several events hinder the efficient operation of the network, such as moving obstacles that block the communication [8]. This demands fast and dynamic association policies that are able to adapt to rough variations in the performance of the network. To address the problem, we propose a lightweight iterative *auction-based approach* that exploits the specific network optimization structure of the problem. We compare our solution method to other association policies present in literature.

The rest of the paper is organized as follows. A description of the system model and the problem formulation is presented in § II. In § III, we describe the solution approach to the multi-assignment problem. In § IV numerical results are presented. Lastly, § V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a mmW network where m access points (APs) can serve n STAs and $n \geq m$. An AP i can serve more than one STA. Every STA j must be associated to just one AP. The set of STAs to which AP i can be assigned is a nonempty set $A(i)$. We introduce the set $B(j)$ as the nonempty set of APs that can serve STA j . A *feasible assignment* S is defined as a set of AP-STA pairs (i, j) , with $j \in A(i)$, where each AP i can be part of more than one pair $(i, j) \in S$, and where every STA j must be part of only one pair $(i, j) \in S$. An illustrative example of an access network is shown in Figure 1. We consider circular coverage areas, where STAs are positioned inside a disc with radius r , centered at the location of AP i .

Every node is equipped with steerable directional antennas (several antenna elements are available in each device) and it can direct its beams to transmit or to receive [7]. We assume that AP i can support its STAs with a separate transmit beam [12]. In particular, in mmW networks beamforming on both the receiver and transmitter side, is used to improve signal quality at reception. As a result of highly directional

transmission and reception, the signal strength is very low at third party STAs that are not involved in the current communication. By choosing for association STAs that are not interfering with each other, an AP can simultaneously transmit to and receive from STAs. We consider the case where all receiver nodes are using single-user detection (i.e., a receiver decodes each of its intended signals by treating all other interfering signals as noise) and the achievable rate from AP i to STA $j \in A(i)$ is

$$R_{ij} = W \log_2 \left(1 + \frac{P_{ij} G_{ij}}{(N_0 + I_j)W} \right), \quad (1)$$

where W is the system bandwidth, P_{ij} is the transmission power of AP i to STA j , G_{ij} is the power gain from AP i to STA j , N_0 is the power spectral density of the noise at each receiver, and I_j is the interference spectral density at STA j . We use the Friis transmission equation together with the *flat-top* transmit/receive antenna gain model [7], [8], where a fixed gain is considered within the beamwidth and zero gain is considered outside the beamwidth of the antenna. In particular, when computing R_{ij} associated with any STA j that resides *inside* the range of AP i , we let $G_{ij} = G_{ij}^{\text{Tx}} G_{ij}^{\text{Rx}} (\lambda/4\pi)^2 (d_{ij}/d_0)^{-\eta}$, where $G_{ij}^{\text{Tx}} > 0$ and $G_{ij}^{\text{Rx}} > 0$ are the transmit and receive antenna gains, respectively, λ is the wave length, d_{ij} is the distance between AP i and STA j , d_0 is the far field reference distance, and η is the path loss exponent. In addition, we consider the well studied 60 GHz characteristics, such as highly directional transmissions with very narrow beamwidths and increased path losses due to the oxygen absorption, in order to assume that the communication interference I_j is very small and does not affect significantly the achievable communication rates in the network [8]. Finally, we assume that for any AP i , the associated backbone capacity L_i is beyond its access network aggregate traffic, i.e., $\sum_{(i,j) \in S} R_{ij} \leq L_i$. We remark that all the assumptions above are natural for 60 GHz [8].

We denote by Q_j the *demanded data rate* of STA j . We define weights $w_{ij} = Q_j / \{(\sum_{k \in A(i)} Q_k) / \|A(i)\|\} = \|A(i)\| Q_j / \sum_{k \in A(i)} Q_k$ as the priority of STA j to select for association the AP i . The general objective is to find a feasible assignment that *maximizes the weighted sum of the STAs throughput*, namely the *total weighted throughput* of the network. Therefore, the association problem is modeled by the following linear optimization problem

$$\max_{x_{ij}} \sum_{(i,j) \in C} w_{ij} R_{ij} x_{ij} \quad (2a)$$

$$\text{s.t. } \sum_{j \in A(i)} x_{ij} \geq 1, \quad \forall i = 1, \dots, m, \quad (2b)$$

$$\sum_{i \in B(j)} x_{ij} = 1, \quad \forall j = 1, \dots, n, \quad (2c)$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in C \quad (2d)$$

where C is the set of all possible AP-STA assignment pairs (i, j) ($S \subseteq C$) and $(x_{ij})_{j \in A(i)}$ are binary decision variables, indicating the STA association. In particular, $x_{ij} = 1$ if STA j is associated to AP i and $x_{ij} = 0$ otherwise, for all i and $j \in A(i)$. (2b) and (2c) ensure that each AP will be assigned to one or more STAs and each STA will be associated to one AP. Note that any feasible point to problem (2) corresponds to a feasible assignment S . Hence the solution to problem (2) corresponds to the optimal feasible assignment S^* . In what follows, we present the proposed solution approach to find S^* .

III. SOLUTION APPROACH

The considered problem (2) is a classical *multi-assignment problem*, where an AP can be assigned to more than one STA. Unfortunately, there are no specialized *network flow methods* that can efficiently solve this class of assignment-like problems. General purpose solution methods such as *primal-simplex*, *primal-dual*, or *relaxation methods* can have high complexity [13]. General methods for linear optimization, such as the simplex or even interior point methods, do not exploit the particular network optimization structure of problem (2) and are not amenable for distributed computation. Thus they are generally less efficient than network optimization methods [14]. Consequently, we resort to network optimization theory and propose a solution method that combines *auction algorithms*. *Auction algorithms* outperform substantially general methods for important classes of problems, both in theory and in practice, and are also naturally well suited for parallel computation, see [14, §4, §7] for a detailed analysis of the criteria to select the best solution approach for assignment problems and a comparison between auction and general purpose polynomial algorithms.

We start by converting problem (2) into a minimum cost flow problem [14] by relaxing (2d) and introducing a virtual supernode s that is connected to each AP i ¹

$$\min_{x_{ij}, x_{si}} \sum_{(i,j) \in C} -w_{ij} R_{ij} x_{ij} \quad (3a)$$

$$\text{s.t. } \sum_{j \in A(i)} x_{ij} - x_{si} = 1, \quad \forall i = 1, \dots, m, \quad (3b)$$

$$\sum_{i=1}^m x_{si} = n - m, \quad (3c)$$

$$\sum_{i \in B(j)} x_{ij} = 1, \quad \forall j = 1, \dots, n, \quad (3d)$$

$$x_{ij}, x_{si} \geq 0, \quad \forall (i, j) \in C \quad (3e)$$

where the sign of the weighted throughput was reversed (cost coefficient) compared to problem (2), *minimization* replaced the *maximization*, and x_{ij} was extended to include also the supernode s . By using the terminology of network optimization, x_{ij} assumes the meaning of amount of flow between i and j . The first constraint ensures that the flow *supply* of each AP i is one unit, whereas the second one declares that s is the source node and the flow that generates is of $n - m$ units. A flow of one unit will reach each STA j . The last constraints declare that the flow of each arc may be infinite, where an arc between i and j denotes the connection (i, j) . A solution to the minimum cost flow problem (3) is identical to the initial multi-assignment problem (2) [14].

By using the duality theory for minimum cost network flow problems [14, §4.2] we formulate the dual problem

$$\min_{\pi_i, p_j, \lambda} \sum_{i=1}^m \pi_i + \sum_{j=1}^n p_j + (n - m)\lambda \quad (4a)$$

$$\text{s.t. } \pi_i + p_j \geq w_{ij} R_{ij}, \quad \forall (i, j) \in C, \quad (4b)$$

$$\lambda \geq \pi_i, \quad \forall i = 1, \dots, m \quad (4c)$$

where $-\pi_i$ is the Lagrangian multiplier associated with constraint (3b) representing the price of each AP i , λ is the Lagrangian multiplier associated with constraint (3c) (i.e., the price of the supernode s), and p_j is the Lagrangian multiplier associated with constraint (3d) (i.e., the price of each STA j). The optimal solution to problem (4) allows us to derive the optimal solution to (2) [14, §4.2, §5].

¹We consider a network where supernode s generates $n - m$ units of traffic and is connected to each AP i by a zero cost arc (s, i) . The traffic that is generated at each AP (*supply*) is of one unit. AP i is connected to STA j by an arc (i, j) with cost $-w_{ij} R_{ij}$.

Algorithm 1 Forward Auction for STA Assignment

Require: Initial values of S, p
Ensure: $w_{ij}R_{ij} - p_j \geq \max_{k \in A(i)} \{w_{ik}R_{ik} - p_k\} - \epsilon, \quad \forall (i, j) \in S$
while there are unassigned STAs **do**
 STA j is unassigned in S
 find the best STA j_i such that:
 $j_i = \arg \max_{j \in A(i)} \{w_{ij}R_{ij} - p_j\},$
 $u_i = \max_{j \in A(i)} \{w_{ij}R_{ij} - p_j\},$
 $\omega_i = \max_{j \in A(i), j \neq j_i} \{w_{ij}R_{ij} - p_j\},$
 if j_i is the only STA in $A(i)$ **then**
 $\omega_j \rightarrow -\infty$
 end if
 $b_{ij_i} = p_{j_i} + u_i - \omega_i + \epsilon = w_{ij_i}R_{ij_i} - \omega_i + \epsilon$
 $p_j = \max_{i \in P(j)} b_{ij},$ where $P(j)$ is the set of APs that STA j
 received a bid,
 remove any pair $(i, j),$ where j was initially assigned to some i under
 $S,$ and add the pair (i_j, j) to S with $i_j = \arg \max_{i \in P(j)} b_{ij}$
end while

To solve problem (4) we need some technical intermediate results. We start by giving the definition of ϵ -Complementary Slackness ($\epsilon - CS$): Let ϵ be a positive scalar, we say that an assignment S and a pair (π, p) satisfy $\epsilon - CS$ if

$$\pi_i + p_j \geq w_{ij}R_{ij} - \epsilon, \quad \forall (i, j) \in C, \quad (5)$$

$$\pi_i + p_j = w_{ij}R_{ij}, \quad \forall (i, j) \in S, \quad (6)$$

$$\pi_i = \max_{k=1, \dots, m} \pi_k, \quad \forall i \text{ s.t. } i \text{ has } > 1 \text{ pair } (i, j) \in S \quad (7)$$

Proposition 1: Consider problems (2) and (4). Let S be a feasible solution for problem (2) and consider a dual variable pair (π, p) . Let $\epsilon < 1/m$ and assume $w_{ij}R_{ij}$ be integer $\forall i, j$. If $\epsilon - CS$ conditions (5), (6), and (7) are satisfied by S and $\pi, p,$ then S is optimal for problem (2).

Proof: The proof is ad-absurdum. Assuming that S is not optimal, then there is a new assignment that can improve the objective function (4) and can give us a new solution: Let E be a cycle, namely a collection of arcs that start and end with the same node, that includes also the supernode s : $E = (s, i_1, j_2, i_2, \dots, i_{k-1}, j_k, i_k, s)$. Here, the nodes i_t represent the APs, and the nodes j_t represent the STAs and $(i_t, j_t) \in S, j_t \in A(i_{t-1}), (i_{t-1}, j_t) \notin S, t = 2, \dots, k$. Based on *max-flow theory* [14, §3], augmentation along E is achieved by replacing $(i_t, j_t) \in S$ by (i_{t-1}, j_t) in $S, t = 2, \dots, k$. AP i_k must be assigned to more than one STAs prior to the previous operation because the arc (i_k, j_k) will exit the assignment and therefore, the AP i_k will be left unassigned. This will result to an infeasible solution to problem (4). Moreover, $k \leq m$ since E cannot contain repeated STAs. Considering also that $\epsilon < 1/m,$ we conclude that $k\epsilon < 1$.

Since we achieved strict cost improvement, we have

$$\sum_{t=2}^k w_{i_t j_t} R_{i_t j_t} + 1 \leq \sum_{t=2}^k w_{i_{t-1} j_t} R_{i_{t-1} j_t}, \quad (8)$$

In order to reveal the $\epsilon - CS$ conditions (5), (6), and (7), we transform (8)

$$\sum_{t=2}^k (w_{i_t j_t} R_{i_t j_t} - p_{j_t}) + 1 \leq \sum_{t=2}^k (w_{i_{t-1} j_t} R_{i_{t-1} j_t} - p_{j_t}). \quad (9)$$

Now using the $\epsilon - CS$ conditions, (9) can be written as

$$\sum_{t=2}^k \pi_{i_t} + 1 \leq \sum_{t=2}^k (w_{i_{t-1} j_t} R_{i_{t-1} j_t} - p_{j_t}) \leq \sum_{t=1}^{k-1} \pi_{i_t} + (k-1)\epsilon. \quad (10)$$

From (10) we have $1 - (k-1)\epsilon \leq \pi_{i_1} - \pi_{i_k}$ which contradicts $k\epsilon < 1,$ because AP i_k is assigned to more than one STAs, i.e., $\pi_{i_k} \geq \pi_{i_1},$ [compare with (7)]. We conclude that our first assumption on that S is not optimal is wrong, which implies that S is optimal. We can get similar results considering that supernode s is not part of $E,$ which completes the proof. ■

Note that, $w_{ij}R_{ij}$ can be rounded to the closest integer

Algorithm 2 Reverse Auction for STA Assignment

Require: $S, (\pi, p)$ and λ from forward auction
Ensure: (1) $\pi_i + p_j \geq w_{ij}R_{ij} - \epsilon, \quad \forall (i, j) \in C$ and (2) $\pi_i + p_j = w_{ij}R_{ij}, \quad \forall (i, j) \in S$
while there are unassociated STAs **do**
 STA j is unassociated in $S,$ find the best AP i_j such that:
 $i_j = \arg \max_{i \in B(j)} \{w_{ij}R_{ij} - \pi_i\},$
 $\beta_j = \max_{i \in B(j)} \{w_{ij}R_{ij} - \pi_i\},$
 $\omega_j = \max_{i \in B(j), i \neq i_j} \{w_{ij}R_{ij} - \pi_i\},$
 if i_j is the only AP in $B(j)$ **then**
 $\omega_j \rightarrow -\infty$
 end if
 $\delta = \min \{\lambda - \pi_{i_j}, \beta_j - \omega_j + \epsilon\}$
 add (i_j, j) to $S: p_j = \beta_j - \delta, \pi_{i_j} = \pi_{i_j} + \delta$
 if $\delta > 0$ **then**
 remove the pair (i_j, j_{old}) where j_{old} was initially assigned to i_j under S
 end if
end while

value. In mmW networks, the effect of rounding influences slightly the true optimal value of Problem (4), because $w_{ij}R_{ij}$ is a large number ($\gg Q_j$) and as a result the fractional part of $w_{ij}R_{ij}$ is relatively smaller than its integer part.

Based on Proposition 1, we present the solution method to problem (4), by an auction mechanism. First, a forward auction algorithm associates each AP to one STA, see Algorithm 1. Then, a modified reverse auction is applied to assign the rest of the STAs to the available APs, see Algorithm 2. Finally, we show that the execution of the two algorithms terminates with an optimal solution by a finite number of iterations².

In particular, we start from a feasible assignment S and the corresponding (π, p) pair that satisfy the first two $\epsilon - CS$ conditions (5), (6). We apply Algorithm 1 until each AP is associated with a single STA and until the $\epsilon - CS$ conditions are satisfied. At this stage, some of the STAs can still be unassigned. We then apply Algorithm 2 that gets as input the assignment achieved by Algorithm 1 (i.e., S and (π, p)). We compute the maximum initial profit for the APs $\lambda = \max_{i=1, \dots, m} \pi_i$. The iterative Algorithm 2 maintains an assignment $S,$ where each AP is associated with at least one STA, and a pair (π, p) that satisfies the first two $\epsilon - CS$ conditions. Algorithm 2 terminates when all unassigned STAs have been assigned to an AP. While λ is kept constant through the execution of Algorithm 2, (7) is satisfied upon termination.

Proposition 2: Consider Algorithms 1 and 2. Let Algorithm 1 run first and then let Algorithm 2 run iteratively. Algorithm 2 terminates in a finite number of iterations bounded by $n^2 \lceil \Delta / \epsilon \rceil,$ where $\Delta = \max_{(i,j) \in C} w_{ij}R_{ij} - \min_{(i,j) \in C} w_{ij}R_{ij}$ with an optimal AP-STA assignment, when $\epsilon < 1/m$.

Proof: In order to prove the optimality and the convergence of the modified reverse auction algorithm we have to show that a) The modified reverse action algorithm iterates by satisfying $\epsilon - CS$ conditions (5), (6), and (7) and $\lambda = \max_{i=1, \dots, m} \pi_i,$ b) The algorithms terminates after a finite number of iterations with a feasible assignment.

The proof of a) is a straight forward application of the well known *theory for auction algorithms* [14, §7] to show that if the $\epsilon - CS$ conditions and $\lambda = \max_{i=1, \dots, m} \pi_i$ are satisfied at the start of an iteration, they are also satisfied at the end.

To show b), we observe that an AP i can receive a bid only

²We assume that a newcomer STA executes the basic RSSI-based association procedure and the auction-based approach is periodically applied, in the background, to optimize the STA assignment. Moreover, the APs utilize the control frames to trigger the initialization of the auction-based assignment approach and to carry the required information to the STAs

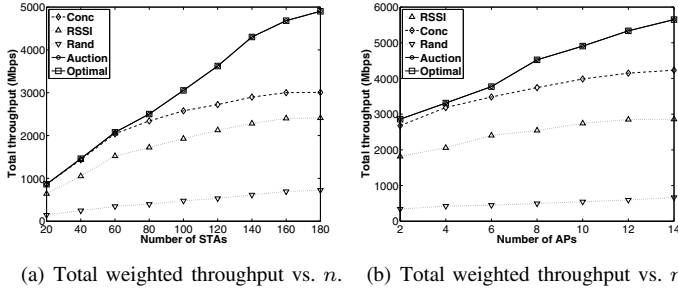


Fig. 2. Simulation results.

a finite number of times after $\pi_i = \lambda$. This is true due to that in each iteration the corresponding STA will be assigned to AP i without changing the association of already assigned STAs to AP i (see Algorithm 2). At the end of each iteration when AP i receives a bid, the profit π_i is either equal to λ or else increases by at least ϵ . Since λ is an upper bound in the profits throughout the algorithm, the main outcome is that each AP can receive a finite number of bids (finite termination).

In the worst case, we consider that all the APs persistently place minimum bid increments ϵ . Then, STA j is initially the most attractive for all APs and it will remain so until its price is increased by at least Δ . This requires at least $\lceil \Delta/\epsilon \rceil$ bids on that STA by every AP and results in a total of $n \lceil \Delta/\epsilon \rceil$ iterations of the algorithm. Proceeding in the same fashion and summing up the total iterations for each stage of the assignment process of n STAs (each bid requires n iterations) we get the upper bound of $n^2 \lceil \Delta/\epsilon \rceil$, which completes the proof. ■

Section IV presents in the sequel the numerical examples.

IV. NUMERICAL ANALYSIS

In this section we present a numerical evaluation study in a multi-user multi-AP environment. We compare the proposed solution approach to a) random association policy (Rand), b) RSSI-based policy (RSSI), which is the standard association mechanism used in 802.11 networks, c) Hop selection metric in [7], used now for association (Conc), and d) optimal solution to problem (2) calculated by IBM CPLEX optimizer (Optimal).

We consider an access network, as depicted in Figure 1. The STAs are uniformly distributed at random. The set of STAs inside the transmission range of AP i is considered as $A(i)$. The set of APs correspond to an overlapping area, in which STA j resides is considered as $B(j)$. When computing G_{ij} , we set $\lambda = 5$ mm, $d_0 = 1$ m, $P_{ij} = P_0 = 0.1$ mW, and $G_{ij}^{\text{Tx}} = G_{ij}^{\text{Rx}} = 1$ without loss of generality³. Moreover, we set $N_0 = -134$ dBm/MHz, $W = 1200$ MHz, and $I_j = 0$, see (1). We assume that Q_{j_s} are uniformly distributed at random on $[0, 0.1]$ Gbps and choose $\epsilon = 0.01$, $L_i = 1$ Gbps. Obstacles block the line-of-site link with probability 0.1 and for a duration of 10 ms. We define the signal to noise ratio (SNR) operating point at a distance d from any AP as

$$\text{SNR}(d) = \begin{cases} P_0 \lambda^2 / (16\pi^2 N_0 W) & d \leq d_0 \\ P_0 \lambda^2 / (16\pi^2 N_0 W) \cdot (d/d_0)^{-\eta} & \text{otherwise} \end{cases}$$

The radius r of each AP is chosen such that $\text{SNR}(r) = 10$ dB. The distance between any consecutive AP is $D = 1.1r$.

³Otherwise, we can let P_0 absorb the resulting multiplicative factor, and hence yield an identical scenario.

Figure 2(a) depicts the total weighted STAs throughput, the main objective of problem (2), achieved by our solution approach in comparison to the other approaches versus n for $m = 10$ APs. Algorithms 1, 2 (Auction) achieve optimal performance and significantly improve the performance of RSSI-based mechanism up to 90% (especially in high load conditions). Figure 2(b) depicts the total weighted STAs throughput versus m for fixed $n = 150$ STAs. The behavior of our approach is similar to Figure 2(a), evincing its optimal and scalable performance. In both figures the metric proposed in [7] achieves worse performance compared to Algorithms 1, 2, since it takes into account the distance and the load of each AP, which does not reflect the dynamic channel variations in presence of blockage or non-line-of-site communication.

V. CONCLUSIONS

We considered the problem of optimizing the allocation of the stations to access points in mmW wireless access networks. The objective was to maximize the total stations's weighted throughput. We presented a solution approach based on auction algorithms. Both theoretical and numerical results evinced the efficient performance of our approach. Thus, it could be well applied in the forthcoming 60 GHz wireless networks.

REFERENCES

- [1] P. Smulders, H. Yang, and I. Akkermans, "On the design of low-cost 60GHz radios for multigigabit-per-second transmission over short distances," *IEEE Commun. Mag.*, vol. 45, no. 12, pp. 44–51, Dec. 2007.
- [2] K. Lee, J. Lee, Y. Yi, I. Rhee, and S. Chong, "Mobile data offloading: how much can WiFi deliver?" *IEEE Trans. Networking*, vol. 21, no. 2, pp. 536–550, Apr. 2013.
- [3] Y. Bejerano, S. Han, and L. Li, "Fairness and load balancing in wireless LANs using association control," *IEEE Trans. Networking*, vol. 15, no. 3, pp. 560–573, June 2007.
- [4] G. Athanasiou, T. Korakis, O. Ercetin, and L. Tassiulas, "Dynamic cross-layer association in 802.11-based mesh networks," in *Proc. 2007 IEEE INFOCOM*, pp. 2090–2098.
- [5] G. Athanasiou, T. Korakis, O. Ercetin, and L. Tassiulas, "A cross-layer framework for association control in wireless mesh networks," *IEEE Trans. Mobile Computing*, vol. 8, no. 1, pp. 65–80, Jan. 2009.
- [6] S. Shakkottai, E. Altman, and A. Kumar, "The case for non-cooperative multihoming of users to access points in IEEE 802.11 WLANs," in *Proc. 2006 IEEE INFOCOM*, pp. 1–12.
- [7] J. Qiao, L. X. Cai, X. S. Shen, and J. W. Mark, "Enabling multi-hop concurrent transmissions in 60 GHz wireless personal area networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3824–3833, Nov. 2011.
- [8] S. Singh, R. Mudumbai, and U. Madhow, "Interference analysis for highly directional 60-GHz mesh networks: the case for rethinking medium access control," *IEEE Trans. Networking*, vol. 19, no. 5, pp. 1513–1527, Oct. 2011.
- [9] A. Xueli, S. Chin-Sean, P. R. Venkatesha, H. Harada, and I. Niemegeers, "Performance analysis of synchronization frame based interference mitigation in 60 GHz WPANs," *IEEE Commun. Lett.*, vol. 14, no. 5, pp. 471–473, May 2010.
- [10] G. M. Olcer, Z. Genc, and E. Onur, "Sector scanning attempts for non-isolation in directional 60 GHz networks," *IEEE Commun. Lett.*, vol. 14, no. 9, pp. 845–847, Sep 2010.
- [11] G. Athanasiou, P. C. Weeraddana, C. Fischione, and L. Tassiulas, "Optimizing client association in 60 GHz wireless access networks," arXiv:1301.2723, 2013. Available: <http://arxiv.org/abs/1301.2723>.
- [12] M. X. Gong, D. Akhmetov, R. Want, and Shiwen Mao, "Multi-user operation in mmwave wireless networks," in *Proc. 2011 IEEE ICC*, pp. 1–6.
- [13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [14] D. P. Bertsekas, *Network Optimization: Continuous and Discrete Models*. Athena Scientific, 1998.