Traffic Engineering
EN3370

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Lecture 8: Modeling Network Traffic I

Why Traffic Modelling?

Logistics of Operating a Network

- demand for services is increasing in an unprecedented manner
- networks require
  - early design
  - planning
  - maintenance
  - continuous development
  - updating

Logistics of Operating a Network

- continual evaluation of networks' performance
- associated methods include
  - measurements
  - analysis
  - modeling
  - subsequent simulations
- traffic modeling plays a pivotal role

Traffic Modeling

- Markov models
- Poisson models
- Bernoulli models
- autoregressive models
- self-similar statistical models
- stochastic processes with long-range dependence
- heavy-tailed distributions

Markov Models

Markov Models

- states are introduced associated with the source
- state of the source can represent
  - its data rate
  - its packet length
- the source changes its characteristic based on the state, e.g.,
  - when the Markov process represents data rate, the source can be in any of several active states and generates traffic with a rate that is defined by the state

Markov Models

- on-off model (just two states)
- models with many states

On-Off Model

- a popular model for bursty sources
- used to describe constant bit rate traffic
- probability that the length of the active period is n time steps is given by the geometric distribution:

$$P[\text{active period} = n] = a^n(1-a)$$

- probability that the length of the silent period is n time steps is given by the geometric distribution:

$$P[\text{silent period} = n] = s^n(1-s)$$

On-Off Model

- state transition diagram?
- illustrate the packet pattern for an on-off source

Models with Many States

- state transition diagram?
- illustrate the packet pattern for an on-off source

Poisson and Bernoulli Models

Poisson and Bernoulli Models

- Poisson:
  - interarrival time between two successive packets/arrivals is exponentially distributed
  - good for describing the arrival of calls/customers at continuous time
- Bernoulli:
  - interarrival time between two successive packets/arrivals is geometrically distributed
  - good for describing the arrival of calls/customers at discrete time
- we learned a lot here!

Autoregressive Models [1]

[1] For standard texts on this, you may refer to § 10-11 of [Storch-1999] *Statistical Analysis in Climate Research*, 1999 by H. von Storch and F. W. Zwiers; § 3 of [Box-2016] *Time Series Analysis Forcasting and Control*, 2016, by G. E. P. Box et al.; § 3 of [Hayes-1996] *Statistical Digital Signal Processing and Modeling*, 1996, by M. H. Hayes.

Autoregressive Models

- a useful and important class of random processes [2]
- can be generated by filtering white noise $v(n)$ with a linear shift-invariant filter, e.g.,

$$x(n) + \sum_{l=1}^{p} a_p(l)x(n-l) = b_0 v(n) \tag{1}$$

- the above process is called an autoregressive process of order p or AR(p).

[2] See § 3.6 of [Hayes-1996]

Yule-Walker Equations

- autocorrelation sequences (i.e., $r_x(k)$) of these processes satisfy a set of equations, i.e.,
  - Yule-Walker equations
  - here the $r_x(k)$s are related to the filter parameters

Yule-Walker Equations

- in particular, by multiplying (1) by $x^*(n-k)$ and taking expectation, we have

$$r_x(k) + \sum_{l=1}^{p} a_p(l) r_x(k-l) = b_0 r_{vx}(k)$$
$$= \sigma_v^2 b_0 h^*(-k)$$

where h is the impulse response of the filter and $\mathbb{E}[v(n)v^*(m)] = \sigma_v^2 \delta(n-m)$

Yule-Walker Equations for AR(p)

- assuming h is causal, Yule-Walker equations for AR(p) process is given by

$$r_x(k) + \sum_{l=1}^{p} a_p(l) r_x(k-l) = \sigma_v^2 |b_0|^2 \delta(k); \quad k \geq 0 \qquad (2)$$

Yule-Walker Equations for AR(p)

- Yule-Walker equations (2) in matrix form:

$$\begin{bmatrix} r_x(0) & r_x(-1) & \dots & r_x(-p) \\ r_x(1) & r_x(0) & \dots & r_x(-p+1) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(p) & r_x(p-1) & \dots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_p(1) \\ \vdots \\ a_p(p) \end{bmatrix} = \sigma_v^2 |b_0|^2 \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- note that the Yule-Walker equations are linear in the coefficients $a_p(k)$

- similarly, we may use the Yule-Walker equations to generate the autocorrelation sequence from a given set of filter coefficients [3]

[3] See pages 112-113 of [Hayes-1996] for an example.

AR(p) for Traffic Modeling

- AR(p) processes can be used to model traffic with short-range dependence
- how?

Autocorrelation function of AR(p)

- the auto-correlation function $r_x(k)$ of a wide-sense stationary AR(p) process can be expressed as [4]

$$r_x(k) = \sum_{l=1}^{p} c(l) y_l^{|k|}, \quad k > 0$$

- y_l, $l = 1, \ldots, p$, are the roots of $\phi(\lambda) = 1 + \sum_{l=1}^{p} a_p(l) \lambda^l$
- $c(l)$, $l = 1, \ldots, p$, can to computed accordingly
- i.e., the autocorrelation function decays exponentially [5]

[4] See § 11.1.9 of [Storch-1999], § 3.2 of [Box-2016].

[5] An example for AP(1) is given in equation (3.122) of [Hayes-1996].

Self-Similar Statistical Models

Key Characteristic

- many models above do not allow descriptions of bursts
- bursts are distinctive in today's network traffic
- bursts can be described by a self-similarity model
- bursts over a wide-range of time scales can be modeled
- correlated with another very important property: long-range dependence
- long-range dependence is also manifested with heavy-tailed probability of density distributions

Key Characteristic

- measure of bursts and self-similarity is explained with the Hurst parameter
- Hurst parameter is correlated with another very important property: long-range dependence
- long-range dependence is also manifested withheavy-tailed probability distributions, e.g.,
  - Pareto's heavy-tailed distribution
  - Weibull's heavy-tailed distribution
- next lecture !