Traffic Engineering
EN3370

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Lecture 1: Overview

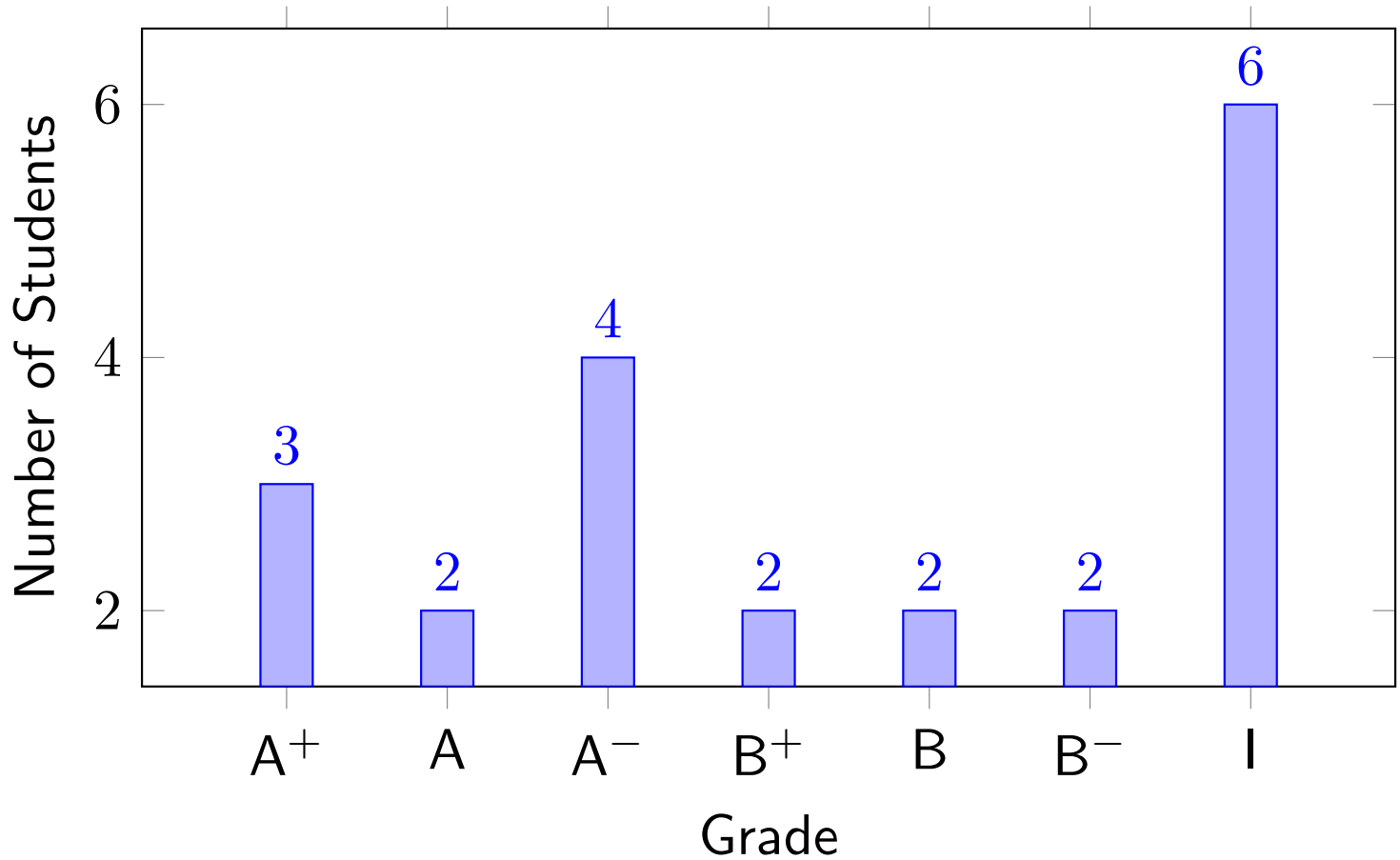
Course Information

- Lecturers: Ajith Pasqual, Chathuranga Weeraddanana
- Lectures: in weekdays, 2 hours/week, 14 weeks
- Office Hours: anytime (e-mails, WhatsApp)

Course Information

- Course Website: moodle of UoM
- Textbooks:
  - indicated in the lecture notes/slides whenever necessary
- Evaluation of Performance:
  - assignments, projects, mid examination: 50%
  - final examination: 50%

Results Distribution of the Last Delivery



What is Traffic

- information transmitted over the networks
- information:
  - telecommunication traffic
  - data communication traffic

Common Components

- input process
  - sequence of requests (from customers/packets) for service
  - inter arrival time
- service mechanism:
  - number of servers
  - time to serve one customer
- queue discipline:
  - disposition of customers who do not have a server
  - leave immediately or wait in a waiting room (e.g., buffer)

What is Traffic Engineering

- traffic theory is defined as the application of probability theory to model
  - traffic
  - resource provisioning
  - traffic management

Tools from Mathematics

- probability
- stochastic processes
- queueing theory
- numerical simulations

Reference Books

- J. M Wozencraft and I. M. Jacobs, Principles of Communication Engineering, 1965 [Wozencraft]
- S. M. Ross, Stochastic Processes, 2nd Edition [Ross-1]
- S. M. Ross, A First Course in Probability, 5th Edition [Ross-2]
- S. M. Ross, Introduction to Probability Models, 9th Edition [Ross-3]
- F. Gebali, Analysis of Computer and Communication Networks [Gebali]
- J. Shortle, et al, Fundamentals of Queueing Theory, 5th Edition [Shortle] § 2
- L. Kleinrock, Queueing Systems, Vol. 1: Theory, 1975 [Kleinrock]

Prerequisites

- It is assumed that the student has been exposed to a first course in probability theory

Some Review of Probability and Random Processes

- Wazencraft, § 2 and § 3
- Ross-1, § 1

Random Processes

- recall.. random variable (RV) X is a rule for assigning to every outcome ω of an experiment a number $x(\omega)$
- A random process $Z(t)$ is a family of time functions depending on the outcome ω
- t usually models time, and any given instant in time is often referred to as an epoch
- the range of t can be restricted as well, e.g., $\{Z(t)\}_{t\in[a,b]}$

RVs from Random Processes

- there is one RV for each epoch t, denoted $Z(t)$
- there must be an underlying sample space Ω over which the RVs are defined [1]
- e.g., for each epoch $t \in \mathbb{R}$, the RV $Z(t)$ is a function $Z(t, \cdot)$ that maps sample points $\omega \in \Omega$ to real numbers

[1] You may refer to § 2.3 of [Wozencraft-1965] for a review of RVs.

Sample Functions from Random Processes

- for a given sample point $\omega \in \Omega$, consider the collection of all the sample values $\{Z(t, \omega)\}_{t \in \mathbb{R}}$
- the collection aforementioned is called a sample function $z(\cdot)$ of the process, where $z : \mathbb{R} \to \mathbb{R}$
- i.e., $z(\cdot)$ is a sample function of the random process $Z(t)$
- draw a picture to visualize RVs and sample functions of a random process

Examples 1

- let $\{Z_k\}_{k\in\mathbb{N}}$ be a sequence of RVs defined on some sample space Ω
- let $\{\phi_n(\cdot)\}_{n\in\mathbb{N}}$ be a sequence of orthonormal real functions
- for each $t \in \mathbb{R}$ define

$$Z(t) = \sum_{k\in\mathbb{N}} Z_k \phi_n(t) \tag{1}$$

- it is clear that random processes can be defined as above

Examples 2

- let $Z_1, Z_2, \ldots$ be a sequence of independent and identically distributed (IID) RVs which take the values ± 1 with probability $1/2$ each
- suppose the modulation pulse $\text{sinc}(t/T)$ is used to yield

$$Z(t) = \sum_{k \in \mathbb{N}} Z_k \ \text{sinc}\left(\frac{t - kT}{T}\right) \tag{2}$$

- at each epoch kT, the RV $Z(kT)$ is simply the binary RV Z_k

Examples 3

- let $\{Z_k\}_{k\in\mathbf{Z}}$ be a sequence of IID zero-mean Gaussian RVs of variance σ^2
- for each $t \in \mathbb{R}$ define

$$Z(t) = \sum_{k\in\mathbf{Z}} Z_k \text{ sinc}\left(\frac{t-kT}{T}\right) \tag{3}$$

- at each epoch kT, the RV $Z(kT)$ is the zero-mean Gaussian RV Z_k

Examples 4

- let Z be a zero-mean Gaussian RVs of variance 1
- for each $t \in [-1, 1]$ define

$$Z(t) = tZ \tag{4}$$

- sample functions are simply straight lines through the origin with random slope

Mean of a Random Process

- let $\{Z(t)\}_{t\in\mathbb{R}}$ be a random process
- the mean at each epoch t

$$\mathbb{E}\left[Z(t)\right] = \bar{Z}(t) \tag{5}$$

  is simply a real-valued function of t
- it is computed directly from the distribution function $F_{Z(t)}(\cdot)$ or density $f_{Z(t)}(\cdot)$

Covariance of a Random Process

- let $\{Z(t)\}_{t\in\mathbb{R}}$ be a random process
- covariance $\mathtt{K}_Z(\cdot,\cdot)$ is a real-valued function of the epochs t and τ
- it is defined by

$$\mathtt{K}_Z(t,\tau) = \mathbb{E}\left[\left(Z(t)-\bar{Z}(t)\right)\left(Z(\tau)-\bar{Z}(\tau)\right)\right] \tag{6}$$

- it is found directly from the joint distribution function $F_{Z(t)Z(\tau)}(\cdot,\cdot)$ or density $f_{Z(t)Z(\tau)}(\cdot,\cdot)$

Stationary Random Processes

- stationary random processes have the property that the location of the time origin is irrelevant
- i.e. they behave the same way at one time as at any other time
- you should have already heard those definitions: see (3.11) of [Wozencraft], § 8.8 of [Ross-1]

Wide-Sense Stationary Random Processes

- let $\{Z(t)\}_{t\in\mathbb{R}}$ be a random process
- suppose $\mathbb{E}[Z(t)] = \bar{Z}(0)$ for all $t \in \mathbb{R}$
- moreover, suppose

$$\mathtt{K}_Z(t,\tau) = \mathtt{K}_Z(t-\tau,0) \quad \text{for all for all} \quad t,\tau \in \mathbb{R}$$

- then the random process $\{Z(t)\}_{t\in\mathbb{R}}$ is said to be wide-sense stationary, see pp 183-184 of [Wozencraft]

Ergodicity of a Random Process

- central problem $\rightarrow$ estimate statistical parameters from real data
- e.g., estimate the mean $\mathbb{E}[Z(t)]$ of a given process $\{Z(t)\}_{t\in\mathbb{R}}$
- for fixed t, $Z(t)$ is a random variable
- suppose we estimate $\mathbb{E}[Z(t)] = \bar{Z}(t)$ by using

$$\hat{\bar{Z}}(t) = \frac{1}{N}\sum_{i-1}^{N} Z(t,\omega_i) \tag{7}$$

Ergodicity of a Random Process

- however, we do not have usually many sample functions as required by (8)
- in practice, usually only a single sample functions of a random process is available
- can we still estimate $\bar{Z}(t)$?
  - possible if $\{Z(t)\}_{t\in\mathbb{R}}$ is stationary, or even wide-sense stationary
  - ergdicity plays a crucial role

Ergodicity of a Random Process

- suppose a stationary process $\{Z(t)\}_{t\in\mathbb{R}}$ with mean $\bar{Z}$ is given
- form the time average of the stationary process $\{Z(t)\}_{t\in\mathbb{R}}$

$$Z_T = \frac{1}{T}\int_{-T}^{T} z(t)dt \tag{8}$$

- thus Z_T is a RV with mean $\bar{Z}$, i.e., unbiased estimator
- Z_T tends [2] to $\bar{Z}$ as $T \to \infty$: $\{Z(t)\}_{t\in\mathbb{R}}$ is mean-ergodic

[2] in some probabilistic sense