

UNIVERSITY OF MORATUWA
Faculty of Engineering
Department of Electronic and Telecommunication Engineering
B.Sc. Engineering
Semester 8 (2016 Batch) Take-Home Examination 1

EN 3370 TRAFFIC ENGINEERING

Time allowed: *Two (24)* hours April 2021

INSTRUCTIONS TO CANDIDATES:

- This assessment contains 4 questions on 5 pages.
- This assessment replaces a part of the regular End-Semester Examination.
- This assessment accounts for **20**% of the module assessment. The total maximum mark attainable is **100**. The marks assigned for each question & sections there of are indicated in square brackets.
- Electronic/communication devices are not permitted. Only equipment allowed is a calculator approved and labeled by the Faculty of Engineering.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, and clearly state it.
- Answer **all** questions.

ADDITIONAL MATERIAL:

- No additional material is provided.

Question 1.

Consider a telecommunication switch modelled as a 10-server Erlang loss system. Measurements show that the offered load a is 4 erlangs.

(a) What percentage of arriving customers are lost? **[5 marks]**

(b) Suppose the rate of service requests will double over the next year. Compute the minimum number of servers that are to be added so that the grade of service of the system will not deteriorate. **[10 marks]**

Question 2.

A telecom service provider offers services that can be modeled as an s-server Erlang loss system. Suppose the arrival rate is 4 customers per minute. The average service time is 1 minute. The service provider earns r units for each customer served. On the other hand, the operating cost of the service provider is c units per server per minute, whether the server is busy or idle.

(a) Determine the profit of the telecom service provider per minute, in terms of s, r, and c. **[5 marks]**

(b) Determine the optimal number of servers, and the corresponding profit in one minute when $r = 2.5$ and $c = 1$. **[10 marks]**

(c) What is the minimum operating cost, denoted $c^\star$, beyond which the service provider cannot yield any positive profit margin. **[10 marks]**

(d) Suppose the operating cost happen to be $c^\star$ units [see part (c) above]. Moreover, suppose that the current values of r cannot be changed. It is assumed that the arrival rate of the customers remains unchanged. However, the service provider is still seeking a strategy to yield a nonzero profit margin. In this respect, explain a potential way to accomplish this goal. **[5 marks]**

Question 3.

Consider the system modeled as a linear birth-death process in which the arrival rate of the customers $\lambda_k = k\lambda$ and the service rate $\mu_k = k\mu$, where $k = 0, 1, \ldots$ and $\mu \neq \lambda$.

(a) Find the partial-differential equation that must be satisfied by $P(z,t)$, which is a time-dependent transform defined as

$$P(z,t) \triangleq \sum_{k=0}^{\infty} z^k P_k(t), \tag{1}$$

where $P_k(t)$ is the probability that number of customers in the system is k at some time t. The notation is similar to those used in the lectures. **[10 marks]**

(b) Assuming that the population size is one at time zero, show that the function that satisfies the equation in part (a) is **[10 marks]**

$$P(z,t) = \frac{\mu\left[1 - e^{(\lambda-\mu)t}\right] - \left[\lambda - \mu e^{(\lambda-\mu)t}\right] z}{\mu - \lambda e^{(\lambda-\mu)t} - \lambda\left[1 - e^{(\lambda-\mu)t}\right] z}.$$

(c) Expanding $P(z,t)$ in a power series determine $P_k(t)$ for all k. **[10 marks]**
[Hint: *The power series expansion of* $A/(1-\alpha z)$ *centered at* 0 is given by $A/(1-\alpha z) = A + A\alpha z + A\alpha^2 z^2 + \cdots$, where A, α are real numbers.]

(d) Determine the mean and variance of the number of customers in the system at time t. **[5 marks]**

Question 4.

Consider a group of s trunks serving *two* types of Poisson traffic. Type 1 is *ordinary* traffic, with arrival rate λ_1 and exponential holding times with mean $(1/\mu_1)$. Type 2 is *wideband* data traffic, with arrival rate λ_2 and exponential holding times with mean $(1/\mu_2)$. Each ordinary call requires m trunks for transmission. If an ordinary call finds less than m idle trunks, it is cleared from the system. On the other hand, each wideband call requires the simultaneous use of n trunks for transmission. If a wideband call finds less than n idle trunks, it is cleared from the system. It is given that $m, n < s$.

(a) The above system can be analyzed by using a discrete-state continuous-time Markov chain. Sketch the state-transition-rate diagram. **[10 marks]**

(b) Determine the balance equations of the state-transition-rate diagram of part (a). **[5 marks]**

(c) What are the proportions of ordinary calls and the wideband calls that are lost? It is sufficient to express the answers in terms of the limiting state probabilities. Exact computation of the limiting state probabilities is not needed. **[5 marks]**

- End of Question Paper -