Traffic Engineering (EN3370) Assignment 2

January 2021

You are supposed to answer the problems listed below. The deadline for the submission of answers: 2021.01.24.

Problem 1

Consider a telecommunication switch which is modelled as an m-server loss system. Arrivals to the switch are Poisson with rate λ and each server has an independently and identically distributed exponential service-time distribution with mean $1/\mu$. The offered load is denoted by a. A statistician observes the system at a random instant and waits until the next customer arrives.

1. Identify the event $\mathcal{E}$ that the customer aforementioned is blocked.
2. Show that the probability $P[\mathcal{E}]$ of the event $\mathcal{E}$ is given by

$$P[\mathcal{E}] = \frac{a}{a+m} B(m,a),$$

where $B(m,a)$ is the well known Erlang's loss formula.

Problem 2

Consider a transmission system consisting of m independent circuits of equal capacity. There are two types (*type* 1 and *type* 2) of sessions arriving the system with Poisson rates λ_1 and λ_2, respectively. The durations (or holding times) of type 1 and type 2 sessions are exponentially distributed with means $1/\mu_1$ and $1/\mu_2$, respectively. The type 1 sessions are *ordinary* sessions whereby each arrival requires *exactly one* of the m servers. If all m servers are busy then any newly arriving session of type 1 is *lost* (or blocked). Each type 2 session requires the *simultaneous* use of m_0 servers. The type 2 session will occupy all m_0 servers simultaneously for the *same* exponentially distributed amount of time whose mean is $1/\mu_2$. If a type 2 session finds less than m_0 idle servers then the session is *lost* to the system.

1. The above system can be analyzed by using a discrete-state continuous-time Markov chain. Identify an appropriate *state description* and sketch the state -transition-rate diagram for $m = 2$ and $m_0 = 2$.
2. By using the balance equations of the state-transition-rate diagram of part (1), determine the limiting state probabilities.
3. Determine the fraction of type 1 sessions and the fraction of type 2 sessions that are lost.

Problem 3

A group of s telephone trunks is offered a_1 Poisson traffic from city 1 and a_2 Poisson traffic from city 2. The call holding times associated with both of the cities are exponentially distributed with common mean $1/\mu$. Thus, note that $a_i = \lambda_i/\mu,\ i = 1, 2$. To give the traffic of city 1 a *higher priority* than that of city 2, any call originated from city 2 that finds less than $n+1$ ($n < s$) trunks idle is cleared (or blocked). However, calls from city 1 are cleared only when all s trunks are busy.

1. Model the system described above as a *birth and death process* and draw the corresponding state-transition-rate diagram.
2. Write the balance equations that determine P_j, the limiting state probability that j calls are in the system ($j = 0, 1, 2, \ldots, s$).
3. Solve the equations for P_j, ($j = 0, 1, 2, \ldots, s$).
4. Write the expressions (in terms of P_j) for the proportions of calls lost in each city.

Problem 4

Consider a call center with only one server. Customer requests are arriving the system with a Poisson rate λ. The service times of requests are exponentially distributed with mean $1/\mu$. Upon arrival, customers are informed their position k of the queue [1] if there are at least one user in the system. As a result, it is noted that the customers become *impatient*. Specifically, when the new arrival finds k in the system, it joins the queue with probability $e^{-\alpha w}$ or leaves with probability $1 - e^{-\alpha w}$, where $w = k/\mu$ and $\alpha \geq 0$.

1. Sketch the state-transition-rate diagram to model the system.
2. Let P_0 denote the probability of finding *none* in the system. In terms of P_0, find the equilibrium probabilities P_k of finding k in the system.

   Hint: You may use that $\sum_{n=1}^{k} n = k(k+1)/2$ *for further simplification of the answer if needed.*
3. Give an expression for P_0 in terms of the system parameters.
4. Suppose $0 < \alpha$, $0 < \mu$. Derive conditions under what the equilibrium solution hold.

   Hint: You may use the fact that series $\sum_{k=0}^{\infty} a_k$ *converges if* $\lim_{k\to\infty} |a_{k+1}|/|a_k| < 1$.
5. When $\alpha \to \infty$, one states that the system resembles the properties of an $M/M/1/1$ queue. By finding P_k explicitly when $\alpha \to \infty$, discuss the validity or the invalidity of the statement aforementioned.

   Hint: Particularized to this problem, you may assume that the limit $\lim_{\alpha\to\infty}$ *and the summation* $\sum_n$ *operations can be interchanged without any technical issues.*

[1] Assume that this operation is completed within 0 seconds.