Mathematical Optimization
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Lecture 4: Math Review: Part 3

Span of a Set of Vectors

- a set of vectors is given: $\{a_1, a_2, \ldots, a_k\}$
- span$\{a_1, a_2, \ldots, a_k\}$ is the set of points that can be hit by any linear combination of the vectors in $\{a_1, a_2, \ldots, a_k\}$
- more specifically,

$$\text{span}\{a_1, a_2, \ldots, a_k\} = \left\{ \sum_{i=1}^{k} \beta_i a_i \;\middle|\; (\beta_1, \beta_2, \ldots, \beta_k) \in \mathbb{R}^k \right\}$$

Span of a Set of Vectors

- illustrate the idea graphically
- exercise:
  - Suppose $a = a_1 + 2a_k$. What can you say about span$\{a_1, a_2, \ldots, a_k\}$ and span$\{a_1, a_2, \ldots, a_k, a\}$? Are they same or different?
  - Suppose a cannot be written as a linear combination of vectors in $\{a_1, a_2, \ldots, a_k, a\}$. What can you say about span$\{a_1, a_2, \ldots, a_k\}$ and span$\{a_1, a_2, \ldots, a_k, a\}$? Are they same or different?

Basis of the Subspace $\mathcal{V}$

- given a subspace $\mathcal{V}$, any set of linearly independent set of vectors $\{a_1, a_2, \ldots, a_k\} \subset \mathcal{V}$ such that

$$\mathcal{V} = \text{span}\{a_1, a_2, \ldots, a_k\}$$

  is called a basis of $\mathcal{V}$

- all bases of a subspace $\mathcal{V}$ contain the same number of vectors

- we call it the dimension of $\mathcal{V}$ or dim $\mathcal{V}$

Basis of the Subspace $\mathcal{V}$

- exercise:
  - you are given a set of vectors $\{a_1, a_2, a_3\}$, where

$$a_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad a_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

  - let $\mathcal{V} = \text{span}\{a_1, a_2, a_3\}$
  - what is $\mathcal{V}$?
  - give a basis for $\mathcal{V}$ by using vectors in $\{a_1, a_2, a_3\}$ itself
  - give a basis for $\mathcal{V}$ without using vectors in $\{a_1, a_2, a_3\}$ itself
  - what is dim $\mathcal{V}$?

Coordinates

- suppose we are given a basis $\{a_1, a_2, \ldots, a_k\}$ of $\mathcal{V}$ and a vector $a \in \mathcal{V}$
- thus, a can be expressed as

$$a = \beta_1 a_1 + \beta_2 a_2 + \cdots + \beta_k a_k \text{ for some } \beta \in \mathbb{R}^k$$

- the coefficients β_i, $i = 1, 2, \ldots, k$ are called the coordinates of a with respect to the basis $\{a_1, a_2, \ldots, a_k\}$

Coordinates

- exercise:
  - consider the basis $\{e_1, e_2\}$ of $\mathbb{R}^2$ and a vector $a = (1, 2) \in \mathbb{R}^2$
  - here $e_1 = (1, 0)$ and $e_2 = (0, 1)$
  - what are the coordinates of a with respect to the basis $\{e_1, e_2\}$?