Mathematical Optimization
YTU 2019

Chathuranga Weeraddana

August 2019

Lecture 3: Math Review: Part 2

Vectors

- a n-column vector is an vertical array of n components, e.g.,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{1}$$

- in this module we always have $x \in \mathbb{R}^n$, i.e., with real components

- in this module if we say x is a n-vector, it means always that x is a n-column vector

Vectors

- an n-row vector is easily represented by using n-vectors, e.g.,

$$[x_1 \; x_2 \; \cdots \; x_n] = x^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}^T$$

Vector Addition

- say $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^n$
- then $c = a + b$ is given by

$$c = a + b = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

- if $a \in \mathbb{R}^m$ and $b \in \mathbb{R}^n$, $m \neq n$, what is $a + b$?

Linearly Independent Set of Vectors

- a set of vectors $\{v_1, v_2, \ldots v_k\}$ is independent whenever

$$\underbrace{(\alpha_1 v_1 + \alpha_2 v_2 + \cdots \alpha_k v_k = 0)}_{A} \implies \underbrace{\alpha_i = 0 \text{ for all } i}_{B}$$

  an equivalent statement:

- no vector v_i can be expressed as a linear combination of the other vectors $v_1 \ldots, v_{i-1}, v_{i+1}, \ldots, v_k$

Linearly Independent Set of Vectors

- a set of vectors $\{v_1, v_2, \ldots v_k\}$ is dependent whenever

  $\text{not}(A \implies B)$, in other words, when $\Big((A) \text{ and } (\text{not } B) \Big)$

  an equivalent statement:

- at least one vector v_i can be expressed as a linear combination of the other vectors $v_1 \ldots, v_{i-1}, v_{i+1}, \ldots, v_k$

Linearly Independent Set of Vectors

- exercise: discuss the linear independence/dependence of following set $\mathcal{S}$:
  - $\mathcal{S} = \{v_1\}$, where $v_1 = (0, 0) \in \mathbb{R}^2$
  - $\mathcal{S} = \{v_1, v_2\}$, where $v_1 = (0, 0) \in \mathbb{R}^2$ and $v_2 = (1, 1) \in \mathbb{R}^2$
  - $\mathcal{S} = \{v_1, v_2\}$, where $v_1 = (2, 2) \in \mathbb{R}^2$ and $v_2 = (1, 1) \in \mathbb{R}^2$
  - $\mathcal{S} = \{v_1, v_2\}$, where $v_1 = (2, 1) \in \mathbb{R}^2$ and $v_2 = (1, 1) \in \mathbb{R}^2$
  - $\mathcal{S} = \{v_1, v_2, v_3\}$, where $v_1 = (1, 0)$, $v_2 = (0, 1)$, and $v_3 = (1, 1)$

Subspace of $\mathbb{R}^n$

- a subset $\mathcal{V} \subseteq \mathbb{R}^n$ is called a subspace of $\mathbb{R}^n$ if $\mathcal{V}$ is closed under the operations of
  - vector addition, i.e.,

$$a, b \in \mathcal{V} \implies (a + b) \in \mathcal{V}$$

    and
  - scalar multiplication, i.e.,

$$a \in \mathcal{V} \implies \alpha a \in \mathcal{V} \text{ for all } \alpha \in \mathbb{R}$$

- exercise: is $0 \in \mathbb{R}^n$ an element of $\mathcal{V}$?
- draw some subspaces to trigger the intuition..