Mathematical Optimization
YTU 2019

Chathuranga Weeraddana

August 2019

Lecture 3: Math Review: Part 1

Places to Read

- for more details see the related sections of [Ref-1] and [Ref-4]

Mathematical Statement

- also called proposition
- it is a sentence that is either true or false
  - 5 is odd is a math. statement which is true
  - 5 is even is a math. statement which is false
  - 5 is cool is not a math. statement

Implications

- many math. statements are written in the form

$$\text{If } A \text{ Then } B, \tag{1}$$

  where A, B both are again math. statements

- the staement in (1) is equivalent to
  - If A is true Then B is true
  - $A \implies B$
  - A is sufficient for B
  - B is necessary for A
  - A only if B
  - not $B \implies$ not A [contrapositive of (1)]

Converse

- converse of $A \implies B$ is

$$B \implies A$$

- suppose we have both $A \implies B$ and $B \implies A$, this means..
  - $A \iff B$
  - A if and only if B
  - A is necessary and sufficient for B
  - B is necessary and sufficient for A

Proof Methods

- let us list 3 most popular ones, in particular for showing $A \implies B$
  - direct method
  - proof by contraposition
  - proof by contradiction
- read Ref-1, section 1.1 (just 2.5 pages) for details

Proof Methods

- consider the math. statement

$$N \in \mathbb{R} \text{ is divisible by } 4 \implies N \in \mathbb{R} \text{ is divisible by } 2 \qquad (2)$$

- such implications are also called conditional statements
- exercise: prove (2) by [1]
  - direct method
  - proof by contraposition
  - proof by contradiction

[1] More easy to understand explanations are available in § 4,5,6 of Ref-4.

Set Definitions

- a set is usually defined using notation as follows: [2]

$$\mathcal{S} = \{x^2 \mid x = 2k, k \in \mathbb{N}\}$$

- $\mathcal{S}$ is the set of all things of the form x^2 such that $x = 2k$ and k is a natural number
- the brace means 'the set of all things of the form'
- vertical line '$|$' means 'such that'; the comma means 'and'
- of course, many more on x can be imposed by adding them with commas, e.g.,

$$\mathcal{S} = \{x \mid x \in \mathbb{R},\ x \geq 0,\ x \leq 5\}$$

[2] See § 1 of Ref-4.

Set Definitions

- consider the following sets in $\mathbb{R}^2$

$$\mathcal{S}_1 = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 = 1\}$$
$$\mathcal{S}_2 = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leq 1\}$$
$$\mathcal{S}_3 = \{(x_1, x_2) \in \mathbb{R}^2 \mid (x_1 - 1)^2 + (x_2 - 2)^2 = 1\}$$
$$\mathcal{S}_5 = \{(x_1, x_2) \in \mathbb{R}^2 \mid (x_1 - 1)^2 + (x_2 - 2)^2 = 1, 2x_1 + x_2 = 1\}$$
$$\mathcal{S}_6 = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + (x_2 - 2)^2 = 4, x_1 \geq 1\}$$
$$\mathcal{S}_7 = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + (x_2 - 2)^2 = 4, x_1 \geq 10\}$$

- exercise: mark those sets in $\mathbb{R}^2$

Quantifiers

- discuss § 2.7 of Ref-4