# Mathematical Optimization
# YTU 2019

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# Lecture 1: Overview

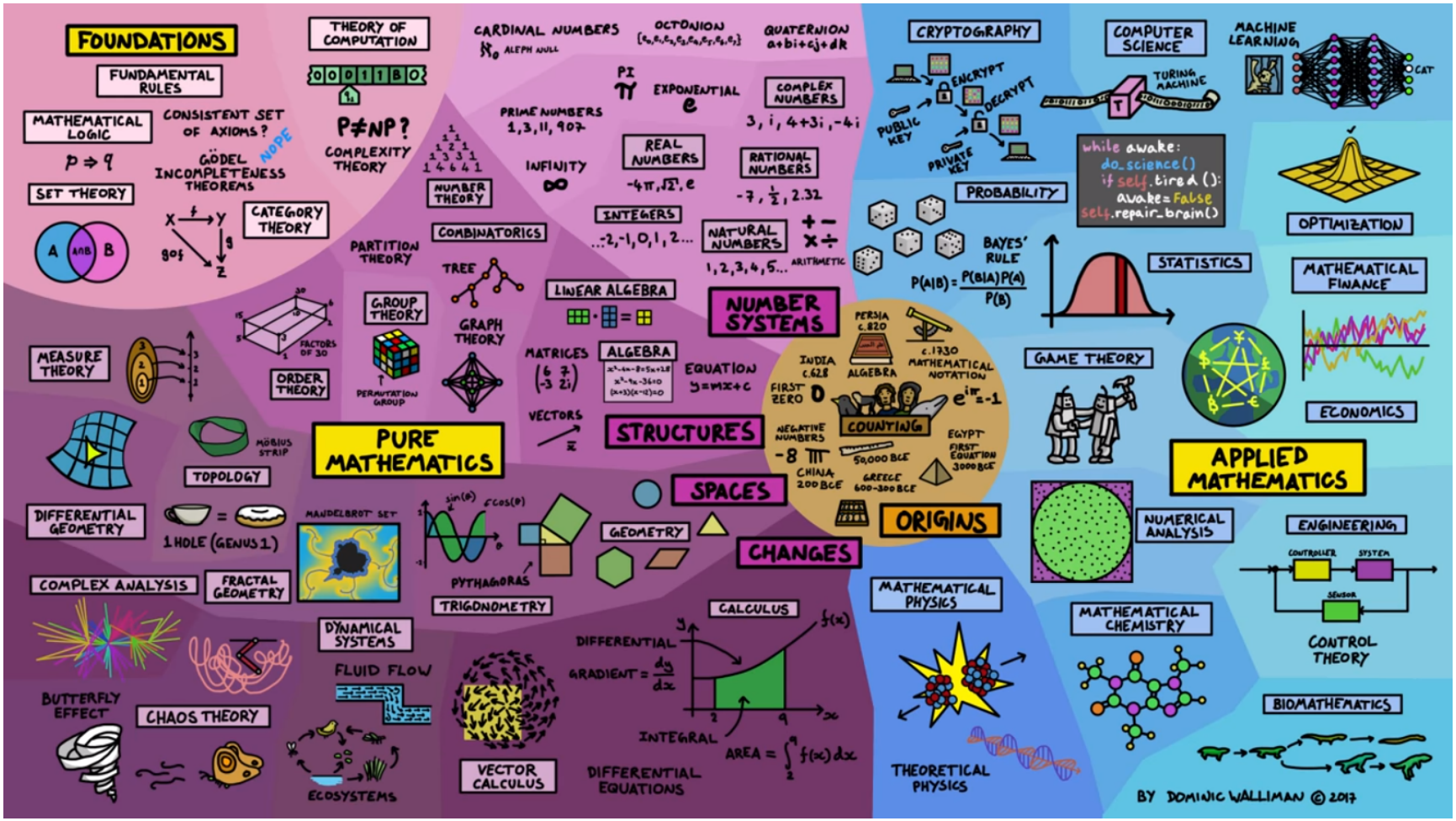
# What is Optimization

- optimization $\rightarrow$ looking for the optimal (best) operating point
- the term best can vary from one entity to another, e.g.,
  - students need to maximize their knowledge in optimization
  - a player need to minimize his careless mistakes
- the term best depends on your objective

# What is Optimization

- the objective is a function of some decision variables
- there are constraints as well
- constraints are again characterized by using functions of the decision variables
  - equality constraints
  - inequality constraints

# Optimization within Map of Mathematics




1

[1] The figure is produced by the video at [Wal09]

# Optimization in Real World

- used in many different fields, e.g.,
  - engineering
  - inventory control
  - transportation
  - economics
  - marketing
  - artificial intelligence
  - machine learning

# Reference Books

- the content discussed in this module is mainly based on the following references:
  - E. K. P. Chong and H. Stanislaw, An Introduction to Optimization, A John Wiley & Sons, Inc., Publication, 2013 [Ref-1]
  - C. D. Mayer, Matrix Analysis and Applied Linear Algebra, SIAM, 2000 [Ref-2]
  - S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004 [Ref-3]
  - R Hammack, Book of Proof, 2009 [Ref-4]

# Notations

- almost always based on [Ref-3, section 1.6]
- depending on the reference, there can be minor changes
- you should be able to pay careful attention

# An Example in Signal Processing

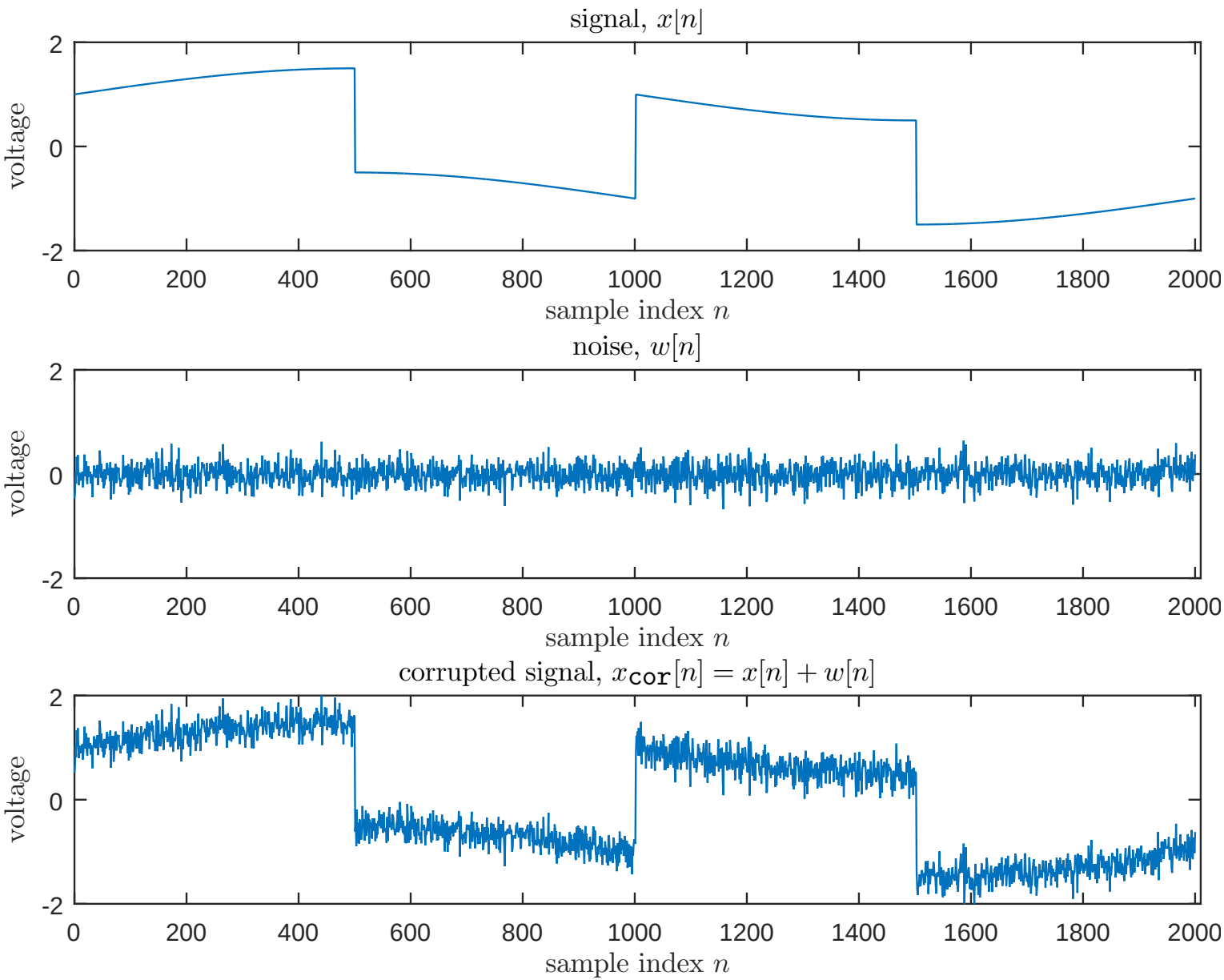
- the example is from Ref-3, § 6.3.3 *Reconstruction, smoothing, and de-noising* [2]
- discrete-time signal is given by $x[n]$, $n = 1, \ldots, N$
- noise is $w[n]$, $n = 1, \ldots, N$
- what is available is the signal $x$ corrupted by an additive noise, i.e.,

$$x_{\texttt{cor}}[n] = x[n] + w[n], \quad n = 1, \ldots, N$$

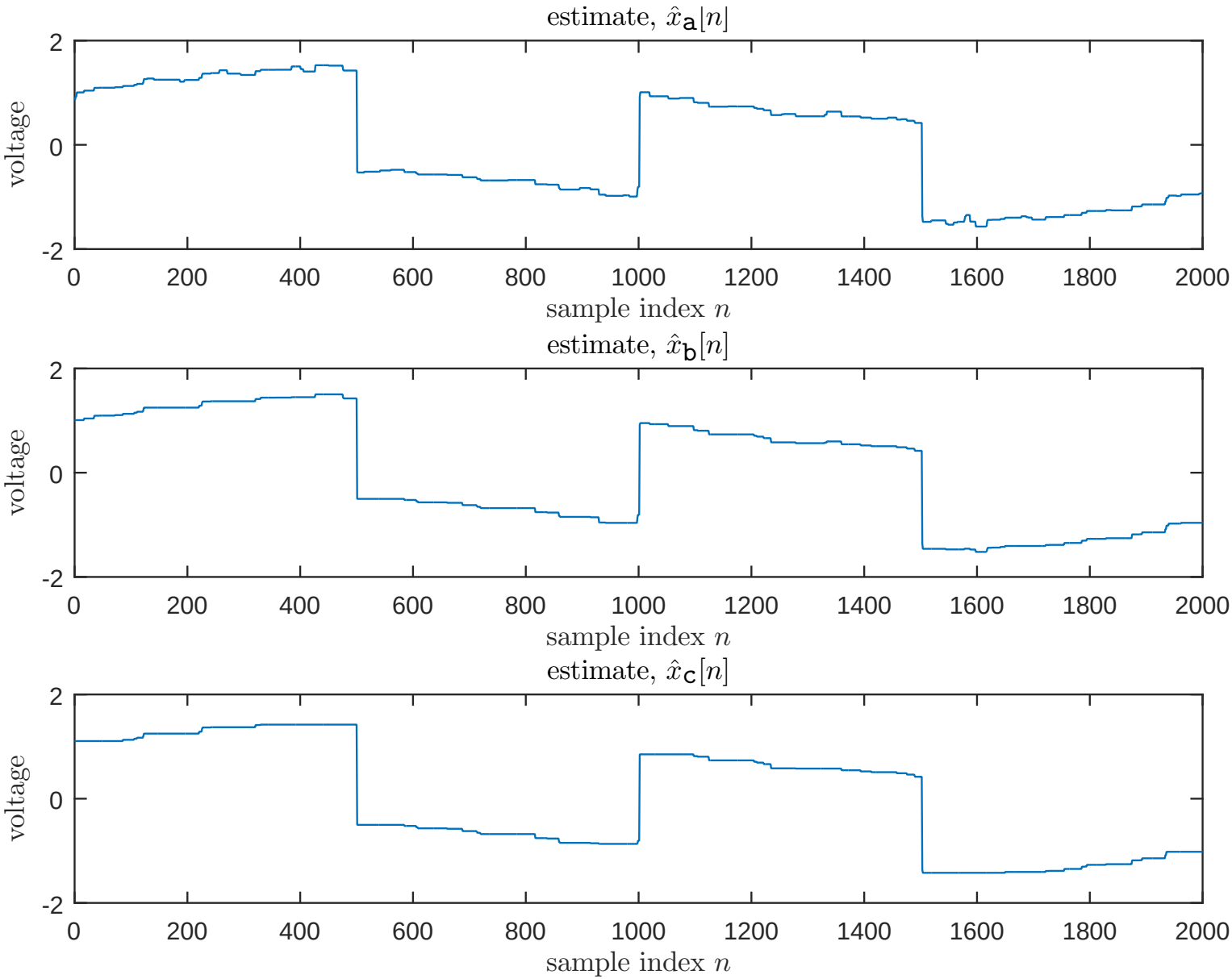
- goal: find an estimate $\hat{x}$ of $x$, given $x_{\texttt{cor}}$

[2] The following plots are adopted from online resources at [Boy18].

# An Example in Signal Processing



# An Example in Signal Processing



# A Toy Problem

- there is a straight road specified by

$$\{(x, y) \in \mathbb{R}^2 \mid x + y = 0\} \tag{1}$$

- a car is moving along this road
- a passenger in the car want to get out of the car at a point P
- P is chosen such that it is the closest (best) point to a shop located at Q $\equiv (0.5, 1)$

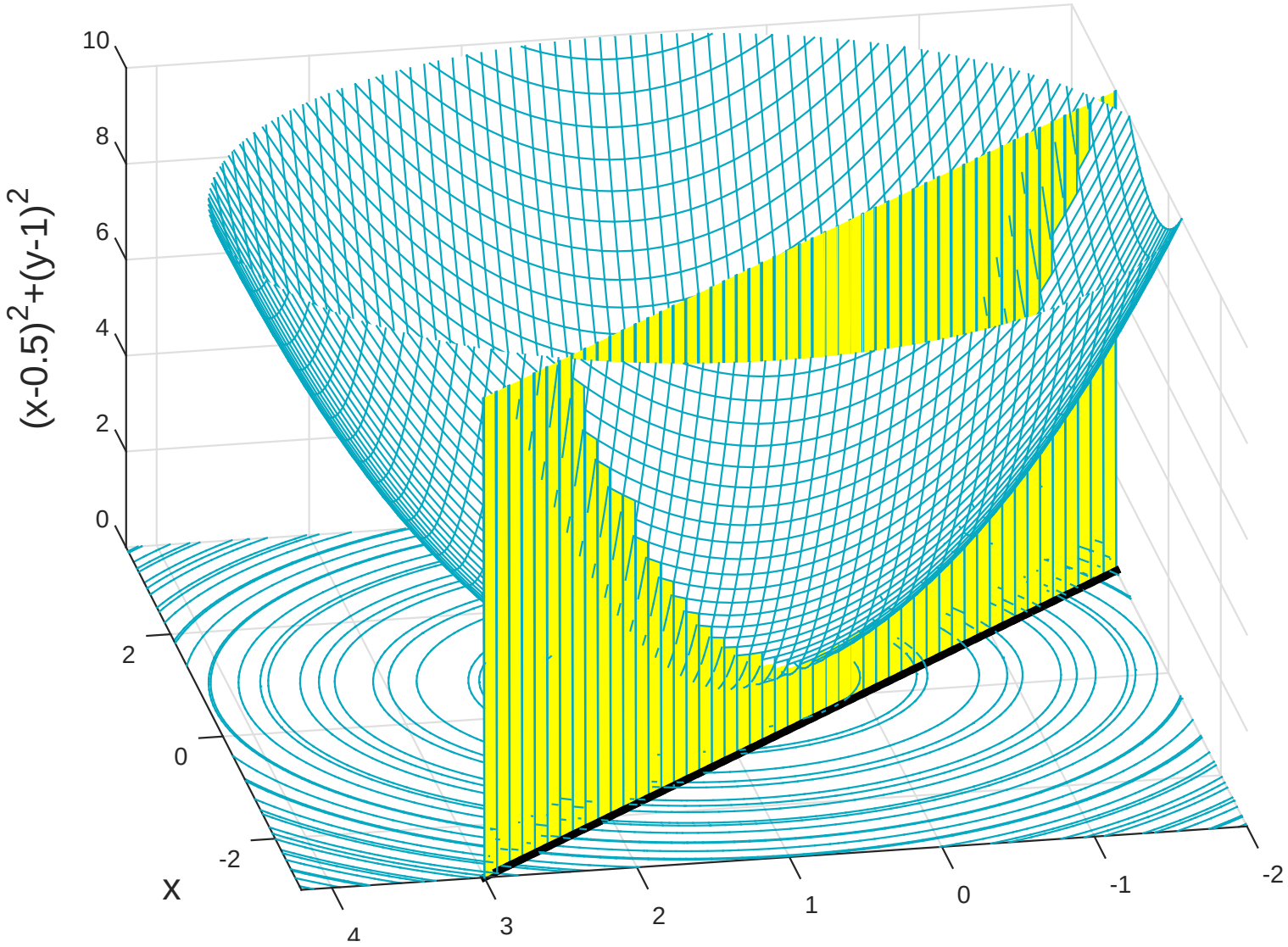
# A Toy Problem

- formulate the task of finding the coordinates of P as an optimization problem

# A Toy Problem

- decision variables ?
- objective function ?
- constraints ?
- optimization problem ?

# Visualize the Toy Problem



# Problem Formulation

- problem can be formulated as

$$\begin{aligned} \text{minimize} \quad & f_0(x, y) \\ \text{subject to} \quad & g(x, y) = 0, \end{aligned}$$

- decision variables: $(x, y)$
- objective function: $f_0(x, y) = (x - 0.5)^2 + (y - 1)^2$
- equality constraint: $g(x, y) = x + y = 0$

# How to Proceed?

- we need to cover some mathematical prerequisites
- e.g., sections 1- 5 of reference 1
  - mathematical statements, sets definitions, quantifiers
  - implications
  - methods of proofs
  - vector spaces and matrices
  - transformations
  - concepts from geometry
  - elements of calculus

# How to Proceed?

- some sections of the prerequisites may be left for you to read
- our main goal is to cover
  - the essential topics of convex analysis: sections 2-5 of reference 3
  - basics of algorithms: sections 9-11 of reference 3
  - use of CVX software for basic problems


S. Boyd.

CVX Example Library.

[Online]. Available: http://cvxr.com/cvx/examples/, 2012, Accessed: 2019.08.18.


Dominic Walliman.

The Map of Mathematics.

[Online]. Available: https://www.youtube.com/channel/UCxqAWLTk1CmBvZFPzeZMd9A, 2017, Accessed: 2018.09.09.