Engineering Decision Theory, Quiz 3

University of Moratuwa, November 2019

Problem 1

Show that the nullspace of a matrix $A \in \mathbb{R}^{m \times n}$ is a subspace.

Problem 2

Show that the range of a matrix $A \in \mathbb{R}^{m \times n}$ is a subspace.

Problem 3

It is often useful to split a vector x into two orthogonal vectors u and v such that $u+v = x$, u is in the direction of a vector w, and v in a direction orthogonal to w. You may assume that all the vectors considered are in $\mathbb{R}^2$. Show that

$$u = \frac{ww^{\mathrm{T}}}{w^{\mathrm{T}}w}\, x \quad \text{and} \quad v = \left(I - \frac{ww^{\mathrm{T}}}{w^{\mathrm{T}}w}\right)\, x,$$

where I is the 2×2 identity matrix. Suppose the matrix $P \in \mathbb{R}^{2\times 2}$ is defined as

$$P = \frac{ww^{\mathrm{T}}}{w^{\mathrm{T}}w}.$$

Show that P is an *idempotent matrix*, i.e., $P^2 = P$. Verify the same for $(I - P)$.