Digital Communications
EN5601

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Lecture 7: Channels, Modulation, and Demodulation (part III)

References

- the discussion is essentially based on [§ 6, Gallager-2008] and [§ 5, Benedetto-2008]

Quadrature Amplitude Modulation (QAM)

- recall: when $x_{\mathrm{BB}}(t)$ is real, frequency is wasted
- QAM solves the frequency waste this problem using a complex baseband waveform

$$x_{\mathrm{PB}}(t) = x_{\mathrm{BB}}(t)e^{2\pi i f_c t} + x^*_{\mathrm{BB}}(t)e^{-2\pi i f_c t} \tag{1}$$

$$= 2\ \mathrm{Re}\left\{x_{\mathrm{BB}}(t)e^{2\pi i f_c t}\right\} \tag{2}$$

$$= 2\ \mathrm{Re}\{x_{\mathrm{BB}}(t)\}\cos(2\pi f_c t) - 2\ \mathrm{Im}\{x_{\mathrm{BB}}(t)\}\sin(2\pi f_c t)$$

Quadrature Amplitude Modulation (QAM)

- at the transmitter:

binary data $\longrightarrow$ symbols $\longrightarrow$ complex signals (constellation) $\longrightarrow$ $x_{\mathrm{BB}}(t) \longrightarrow x_{\mathrm{PB}}(t)$

- at the reviver:

$x_{\mathrm{PB}}(t) \longrightarrow x_{\mathrm{BB}}(t) \longrightarrow$ complex signals (constellation) $\longrightarrow$ symbols $\longrightarrow$ binary data

Complex (QAM) Signal Set

- recall some notations
- R is data rate in bits per second
- $M = 2^b$ symbols (binary b-tuples)
- map M symbols to M signals, signal rate is $R_s = R/b$ signals per second
- signal interval is $T = 1/R_s$ seconds
- signals u_k are complex numbers (real 2-tuples)
- signal set $\mathcal{A}$ is the constellation of M complex numbers

Standard $\sqrt{M} \times \sqrt{M}$-QAM Signal Set

- it is the Cartesian product of two $\sqrt{M}$-PAM sets
- i.e.,

$$\mathcal{A} = \left\{ a + ib \;\middle|\; a, b \in \tilde{\mathcal{A}} \right\} \tag{3}$$

where

$$\tilde{\mathcal{A}} = \left\{ \frac{-d(\sqrt{M}-1)}{2}, \frac{-d(\sqrt{M}-3)}{2}, \cdots, \frac{d(\sqrt{M}-3)}{2}, \frac{d(\sqrt{M}-1)}{2} \right\}$$

- energy per signal E_s is

$$\begin{aligned} E_s &= 2 \times (\text{energy per } \sqrt{M}\text{-PAM signal}) \\ &= 2 \times \frac{d^2[(\sqrt{M})^2 - 1]}{12} \\ &= \frac{d^2[M-1]}{6} \end{aligned}$$

Signals to Complex Waveforms

- Nyquist theory works without change, modulation pulses can be complex
- band-edge symmetry requires real $g(t)$
- more importantly, orthogonality of $p(t - kT)$ requires $g(t)$ to be real, but not $p(t)$
- nominal passband Nyquist BW is $(1/T)$, actual passband BW is little more

QAM Implementation

- suppose $p(t)$ is real and the baseband representation is

$$x_{\mathrm{BB}}(t) = \sum_k u_k p(t - kT)$$

where $u_k \in \mathcal{A}$, see (3)

- then

$$\mathrm{Re}\left\{x_{\mathrm{BB}}(t)\right\} = \sum_k \mathrm{Re}\{u_k\} p(t - kT)$$

$$\mathrm{Im}\left\{x_{\mathrm{BB}}(t)\right\} = \sum_k \mathrm{Im}\{u_k\} p(t - kT)$$

QAM Implementation

- thus, the real passband signal is given by

$$x_{\mathrm{PB}}(t) = 2\left(\sum_k a_k p(t-kT)\right)\cos(2\pi f_c t) - 2\left(\sum_k b_k p(t-kT)\right)\sin(2\pi f_c t)$$

- draw a diagram of the transmitter

Choosing a Good Signal Set

- one needs to choose M points each is at a distance at least d to others
- meanwhile the energy of the signal set should be minimized
- for the most part, standard signal sets are used
- however, a number of signal sets or constellations emerged