# Digital Communications
# EN5601

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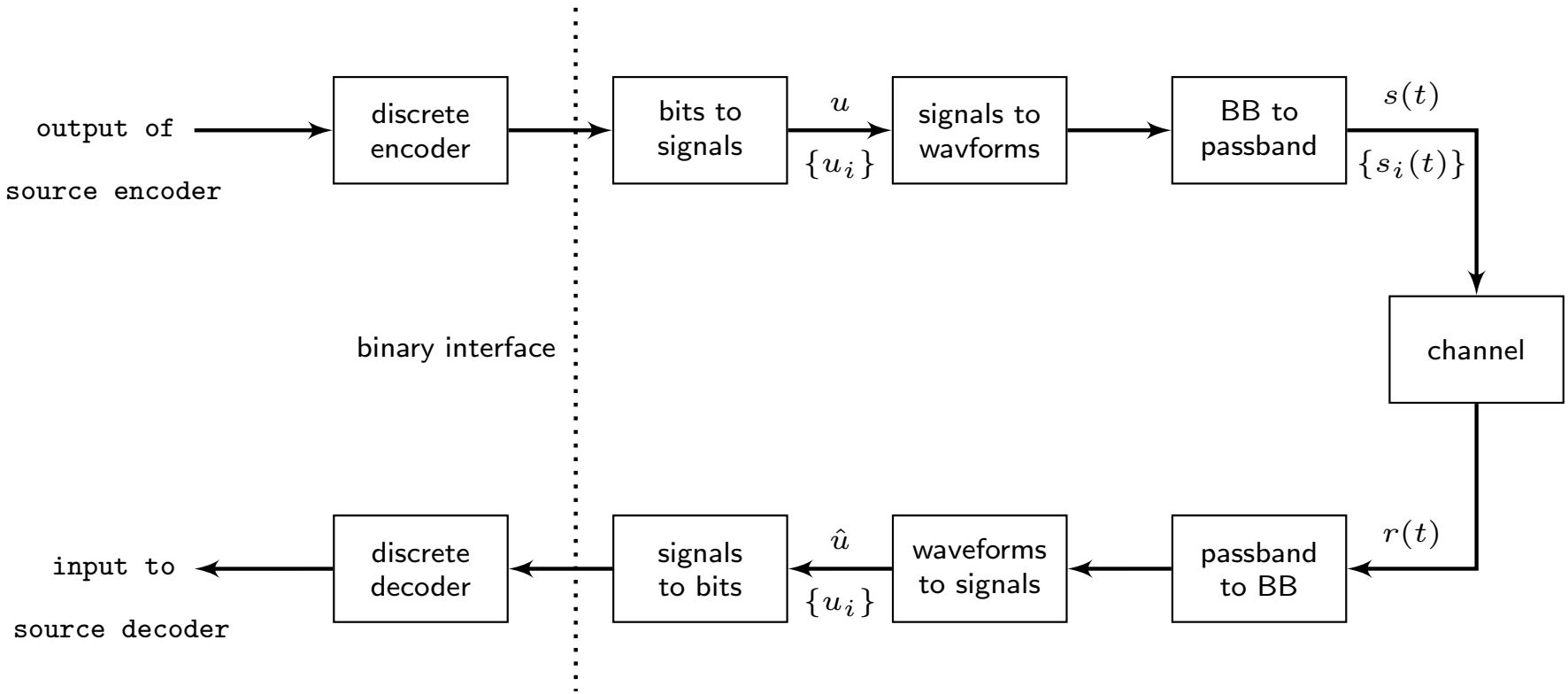
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# Lecture 13: Receiver Implementation

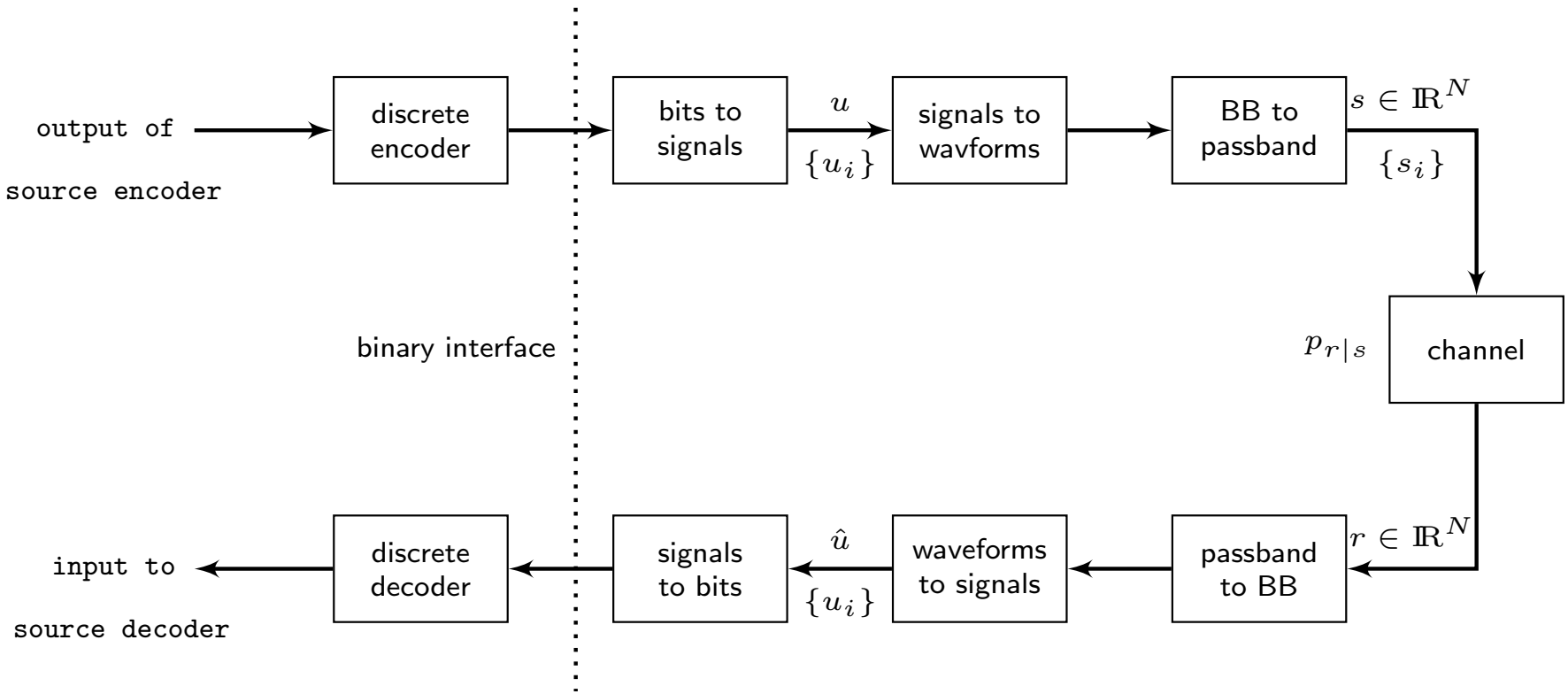
# References

- the discussion is essentially based on [§ 4, Wozencraft-1965], [§ 4.3, Bigliery-2008], [§ 5 Wozencraft-1965]

# Waveform Channel



# Vector Channel



# Vector AWGN Channel

- noise vector is $w \in \mathbb{R}^N$ is given by

$$w = (w_1, w_2, \ldots, w_N)$$

- the receive vector is $r \in \mathbb{R}^N$ and signal vector $s \in \mathbb{R}^N$ are related by

$$r = s + w$$

# Functions of the Optimum Receiver

- first, the relevant data vector is calculated, i.e.,

$$r = (r_1, r_2, \ldots, r_N),$$

where

$$r_j = \int_{-\infty}^{\infty} r(t)\varphi_j(t)\ dt, \qquad j = 1, 2, \ldots, N$$

# Functions of the Optimum Receiver

- then, the receiver sets $\hat{u}(t) = u_k$ if the decision function [1]

$$||r - s_i||^2 - N_0 \ln P[u_i]$$

is minimum for $i = k$

[1] We use that $2\sigma^2 = N_0$.

# Functions of the Optimum Receiver

- in practice, squares are avoided by noting that

$$||r - s_i||^2 = ||r||^2 + ||s_i||^2 - 2\ r^{\mathsf{T}} s_i$$

- since $||r||^2$ is independent of $i$, the aforementioned decision rule is equivalent to: set $\hat{u}(t) = u_k$ if

$$r^{\mathsf{T}} s_i + \underbrace{\frac{1}{2}\left(N_0 \ln\ P[u_i] - ||s_i||^2\right)}_{c_i}$$

  is maximum for $i = k$

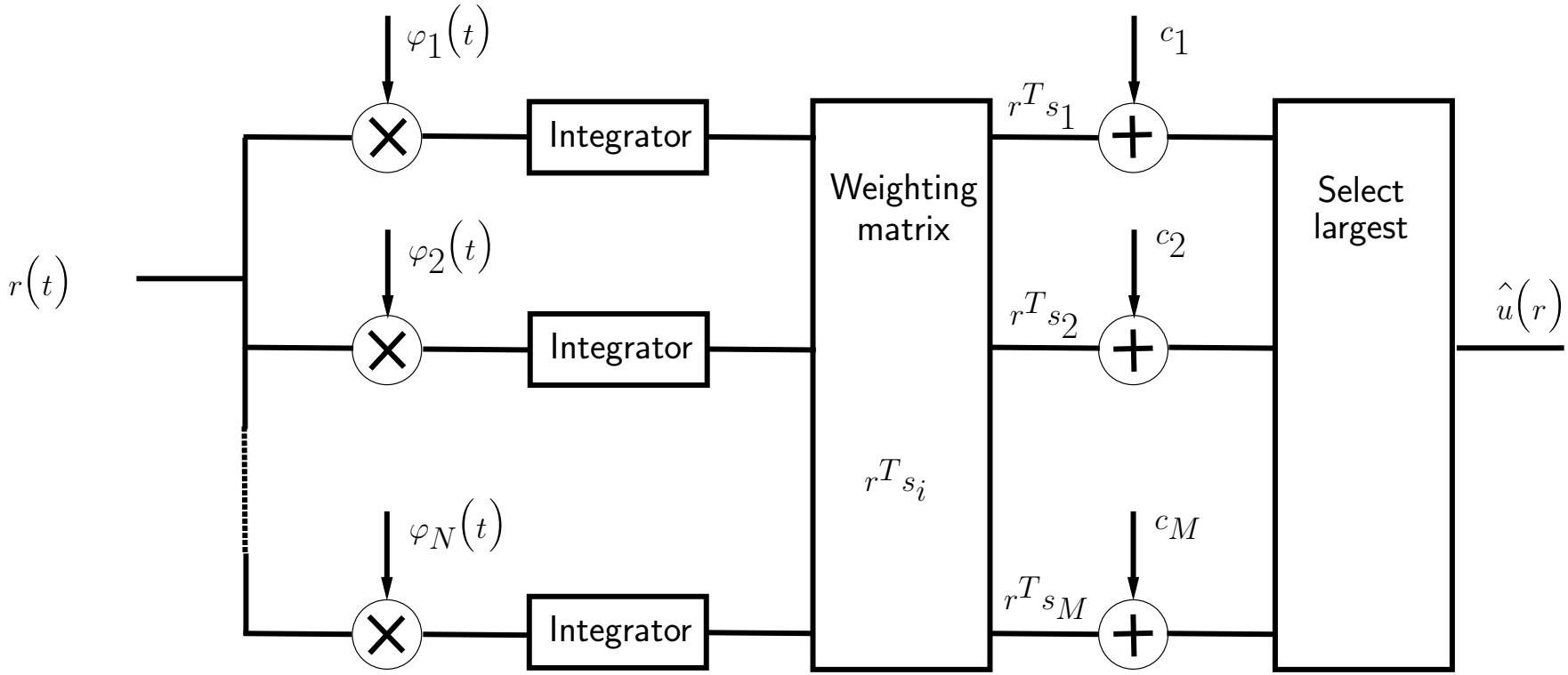
- so computation of the inner product is needed

# Correlator Receiver

# Correlator Receiver

- the relevant received vector $r$ is obtained from the received waveform by a bank of $N$ multipliers
- inner product $r^{\mathsf{T}} s_i$ is computed by using analog computation ($M$ is not large) of by digital computation ($M$ is very large)

# Correlator Receiver

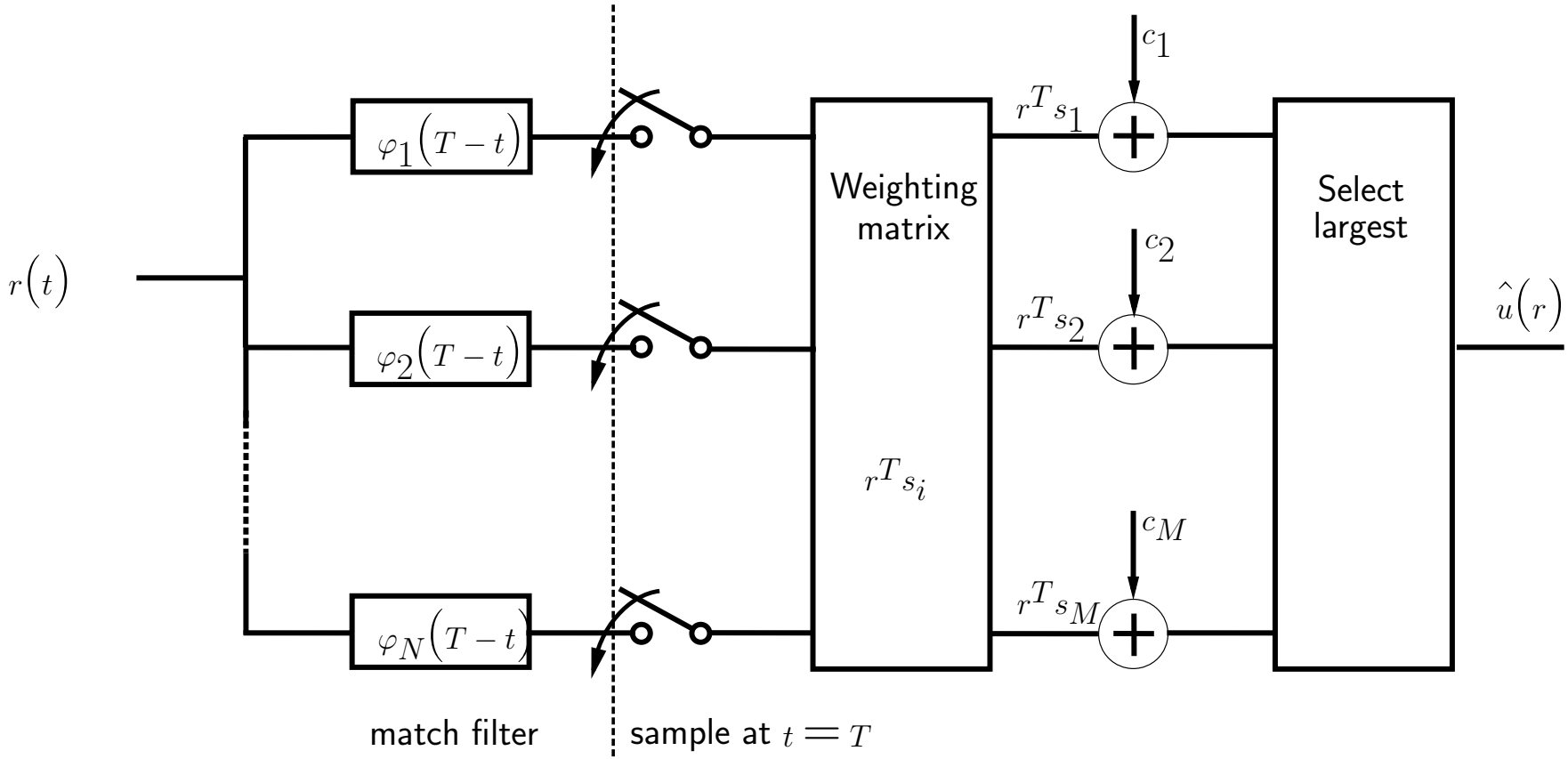


# Matched Filter Receiver

# Matched Filter Receiver

- each member of orthonormal basis $\{\varphi_i\}$ is identically zero outside some finite interval $[0, T]$
- the use of multipliers can be avoided
- desirable because accurate analog multipliers are hard to build

# Matched Filter Receiver



# Matched Filter Receiver

- consider, for instance, the output $u_j(t)$ of the linear filter with impulse response $h_j(t) = \varphi_j(T - t)$
- filter input is $r(t)$
- we have

$$\begin{aligned} u_j(t) &= \int_{-\infty}^{\infty} r(\lambda) h_j(t - \lambda) d\lambda \\ &= \int_{-\infty}^{\infty} r(\lambda) \varphi_j(T - t + \lambda) d\lambda \end{aligned}$$

# Matched Filter Receiver

- thus, sampled filter output $u_j(T)$ is given by

$$\begin{aligned} u_j(T) &= \int_{-\infty}^{\infty} r(\lambda)\varphi_j(\lambda)d\lambda \\ &= r_j \end{aligned}$$

- as a result, $r \in \mathbb{R}^N$ is available and the decision rule can be implemented

# Matched Filter Receiver (Matching Directly to the Signals)

# Inner Product $r^{\mathsf{T}} s_i$

- recall: the decision rule is to: set $\hat{u}(t) = u_k$ if

$$r^{\mathsf{T}} s_i + c_i$$

  is maximum for $i = k$

- so computation of the inner product is needed

# Inner Product $r^{\mathsf{T}} s_i$

- let us see what is $r^{\mathsf{T}} s_i$
- we have

$$
\begin{aligned}
r^{\mathsf{T}} s_i &= \sum_{j=1}^{N} r_i s_{ij} \\
&= \sum_{j=1}^{N} \left( \int_{-\infty}^{\infty} r(t) \varphi_j(t) \, dt \right) s_{ij} \\
&= \int_{-\infty}^{\infty} r(t) \left( \sum_{j=1}^{N} \varphi_j(t) s_{ij} \right) \, dt \\
&= \int_{-\infty}^{\infty} r(t) s_i(t) \, dt
\end{aligned}
$$

# An Equivalent Decision Rule

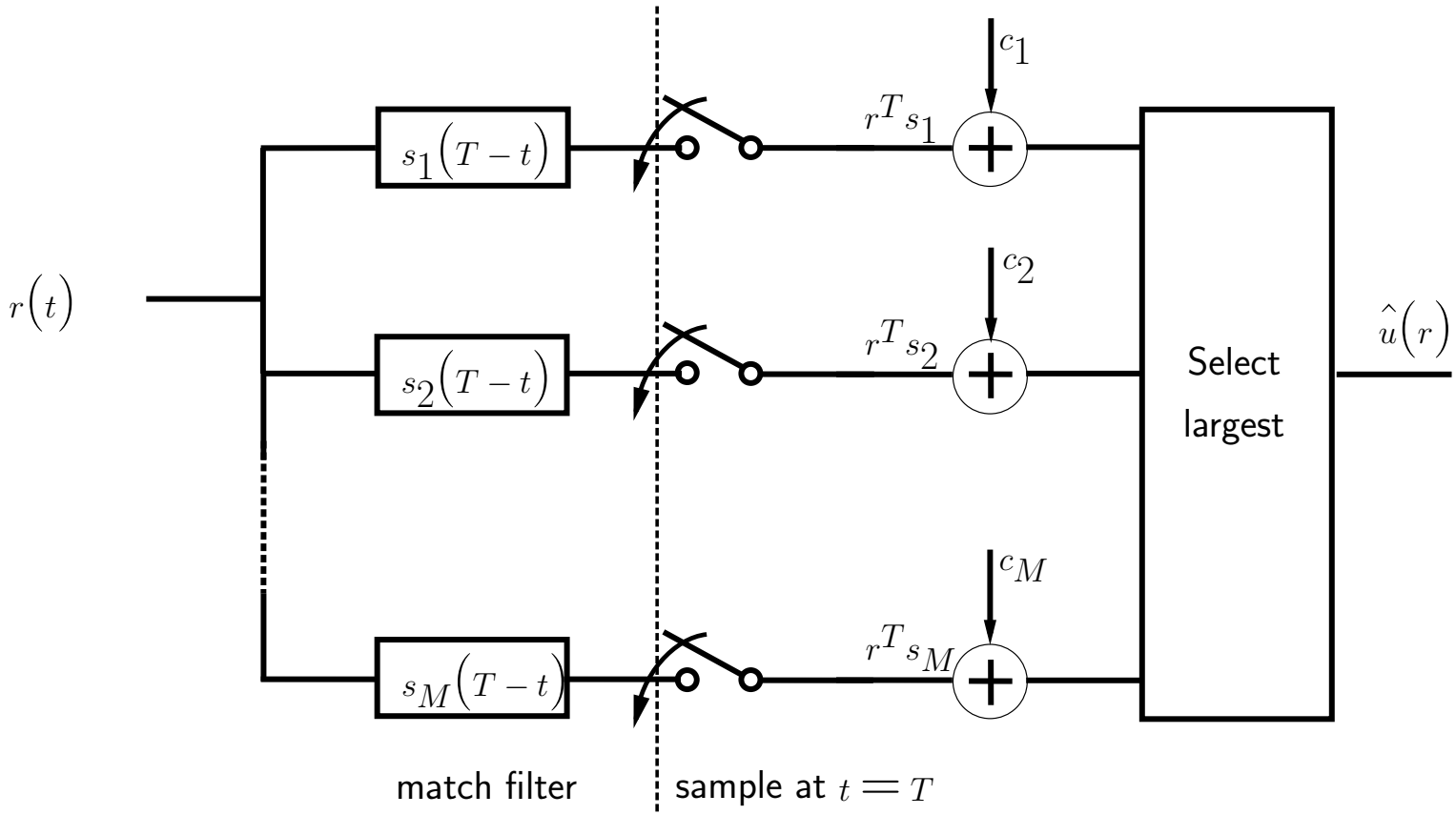
- the decision rule afore mentioned is equivalent to: set $\hat{u}(t) = u_k$ if

$$\int_{-\infty}^{\infty} r(t)s_i(t)\ dt + c_i$$

  is maximum for $i = k$

- thus, the correlator receiver or the match filter receiver can also be implemented by using the above identity

# Receiver Matched Directly to the Signals



# Receiver Matched Directly to the Signals

- $M$ matched filters are required compared to the previous design where $N$ matched filters are used [2].
- however, it is well known that $N \leq M$
- when $M \gg N$, which is usually the case in practice $\rightarrow$ less expensive to go for the match filter receiver or the correlator receiver with $N$ elements

[2] Same arguments hold also for the equivalent correlator receiver with $M$ correlators instead of $N$.