


**UNIVERSITY OF MORATUWA**
Faculty of Engineering
Department of Electronic and Telecommunication Engineering
PG Diploma/M.Sc. in Telecommunications 2020
Final Assessment

**EN 5601 DIGITAL COMMUNICATIONS**

Time allowed: *One (1)* hours January 2022

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**INSTRUCTIONS TO CANDIDATES**:

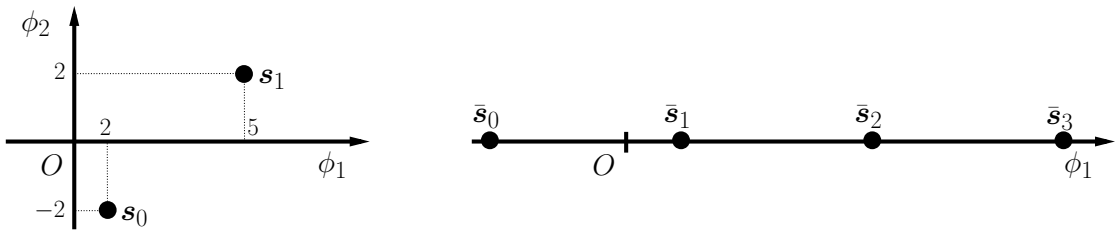
- This paper contains 2 questions on 3 pages.
- This examination accounts for **30**% of the module assessment. The total maximum mark attainable is **50**. The marks assigned for each question and sections there of are indicated in square brackets.
- This is an **open-book** examination.
- Electronic/communication devices are not permitted. Only equipment allowed is a calculator approved and labeled by the Faculty of Engineering.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, and clearly state it.
- All questions carry equal marks.
- Answer **all** questions.

**ADDITIONAL MATERIAL**:

- No additional material is provided.

**Question** 1.

Figure Q1(a) shows signals $\boldsymbol{s}_0$ and $\boldsymbol{s}_1$ that correspond to two equally likely messages. Figure Q1(b) shows signals that correspond to four equally likely messages, where the distance between neighboring signals is $4\sqrt{2}$. In each case, the messages are to be transmitted over an additive white Gaussian noise (AWGN) channel.




(a) (b)

Figure Q1: Vector representation of signal sets: (a) $\mathcal{S}$; (b) $\bar{\mathcal{S}}$.

(a) Compute the average energy $E_{\mathcal{S}}$ of the signal set $\mathcal{S} = \{\boldsymbol{s}_0, \boldsymbol{s}_1\}$. **[5 marks]**

(b) Sketch a minimum energy configuration $\mathcal{S}_{\texttt{min}}$ of $\mathcal{S}$. Compute the corresponding minimum energy $E_{\texttt{min}}$. **[5 marks]**

(c) Let $P_{\mathcal{S}}$, $P_{\mathcal{S}_{\texttt{min}}}$, and $P_{\bar{\mathcal{S}}}$ denote the probability of error when the AWGN channel is used with the signal sets $\mathcal{S}$, $\mathcal{S}_{\texttt{min}}$, and $\bar{\mathcal{S}}$, respectively.

i. Compute $P_{\mathcal{S}_{\texttt{min}}}$ in terms of $P_{\mathcal{S}}$. **[2 marks]**

ii. Determine $P_{\bar{\mathcal{S}}}$ in terms of $P_{\mathcal{S}}$. **[3 marks]**

(d) Suppose we require $P_{\bar{\mathcal{S}}} = P_{\mathcal{S}_{\texttt{min}}}$. If you are to accomplish the requirement, explain appropriate modifications that the signal set $\bar{\mathcal{S}}$ should undergo. **[10 marks]**

**Question** 2.

Symbol error probability $P(e)$ of two $M$-PAM transmission schemes are plotted in Figure Q2. Symbols are transmitted over an AWGN channel, where the noise power spectral density $N_0/2 = 0.25$ $\mu$W/Hz. . The transmissions are to be designed such that the symbol error probability $P(e)$ is no greater than $10^{-6}$, i.e., $P(e) \leq 10^{-6}$.

(a) Determine the minimum energy per bit $E_2$ and $E_8$ in J/bit for $M = 2$ and $M = 8$, respectively, if the symbol error probability constraint $P(e) \leq 10^{-6}$ is to be satisfied. **[5 marks]**

Hint: You may use Figure Q2 or inverse $Q$ function values: $Q^{-1}(10^{-6}) = 4.7534$ and $Q^{-1}((4/7)10^{-6}) = 4.8653$.

(b) Suppose a transmission rate of 1000 bits/s is to be obtained with $P(e) \leq 10^{-6}$. Compute the required average power $P_2$ and $P_8$ in mW for $M = 2$ and $M = 8$, respectively. **[10 marks]**

(c) Compare and contrast trade-offs using a 2-PAM system with $P(e) = 10^{-6}$ and an average power $P_2$ as supposed to using an 8-PAM system with $P(e) = 10^{-6}$ and an average power $P_8$. Arguments and justifications for your answer are to be clearly pointed. **[10 marks]**

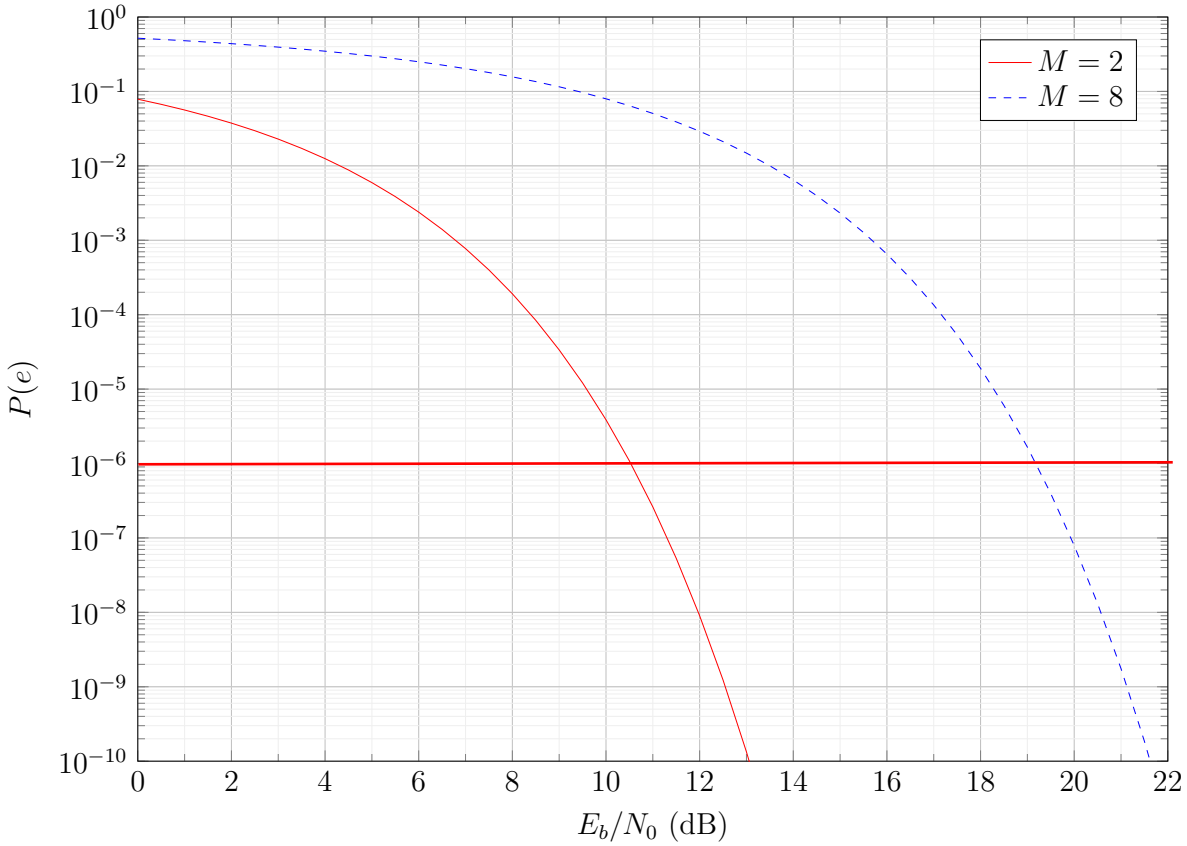



Figure Q2: Symbol Error Probabilities of $M$-PAM.