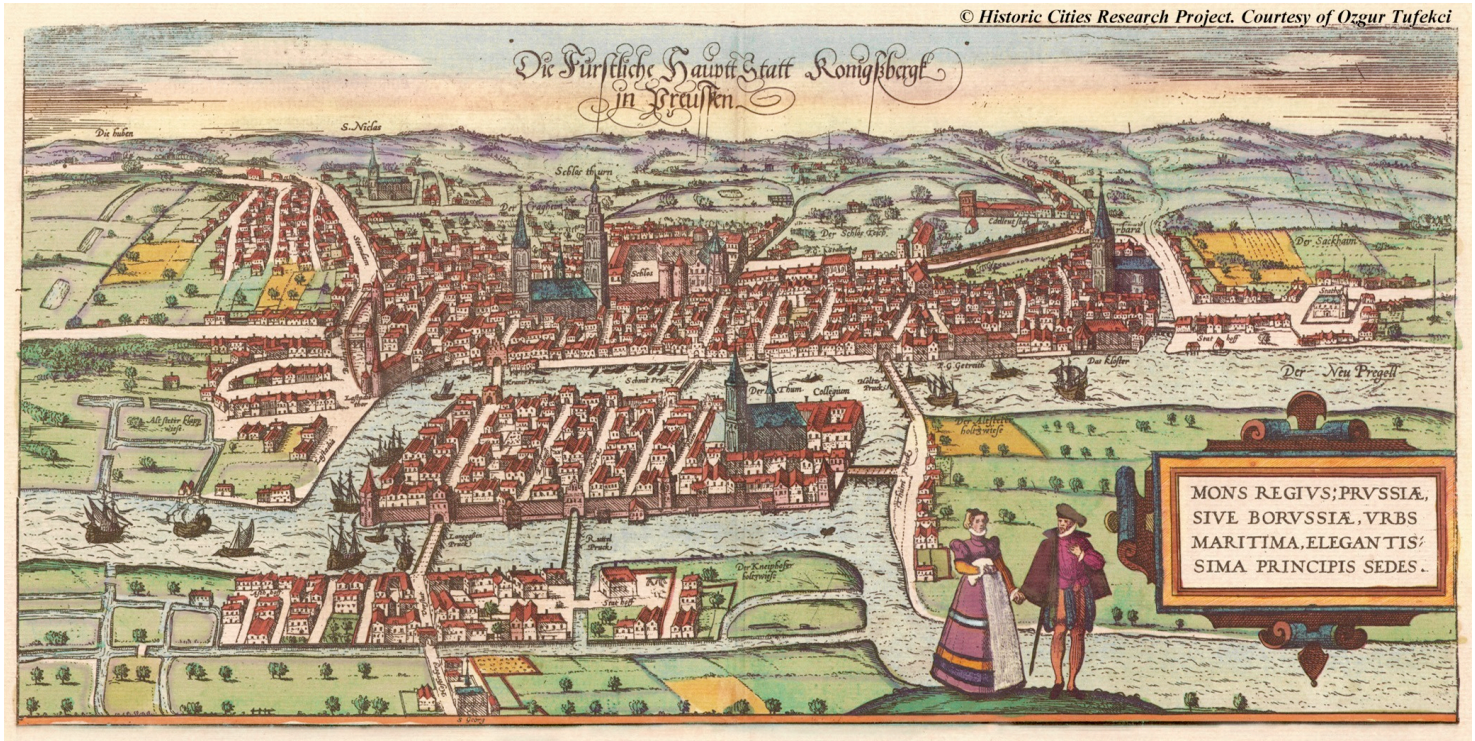
Network Optimization



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Winter 2014
Course code: FEL3250

Lecture 2 – 30-01-2014

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Previous lecture

- Network Optimization
- Graphs, paths, cycles, flows, arcs
- Minimum Flow Problem
- Solution algorithms
- Basic optimality condition

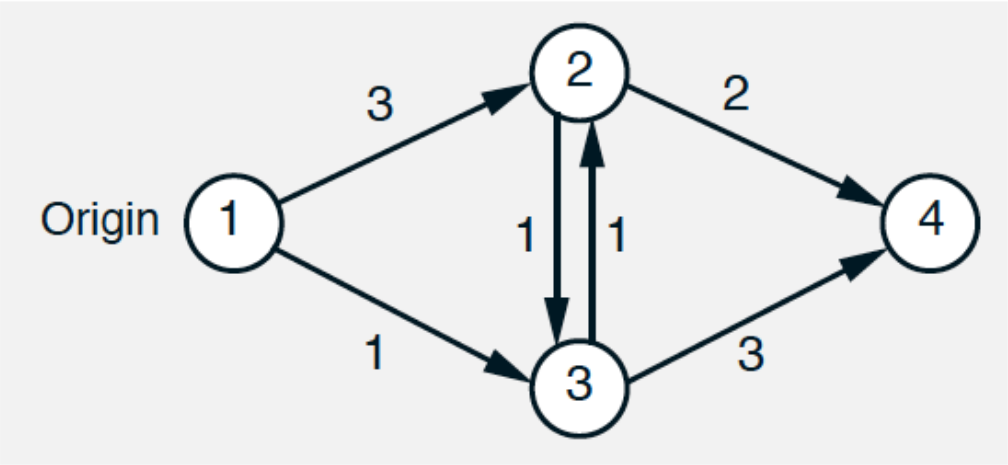
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Today's lecture

- What is the Shortest Path Problem?
- What is the generic Shortest Path Algorithm?
- What is a Label Setting Method (Dijkstra Algorithm)?
- What is a Label Correcting Method (Bellman-Ford Algorithm)?

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Example of GSPA: Figure 2.3



Iteration #	Candidate List V	Node Labels	Node out of V
1	$\{1\}$	$(0, \infty, \infty, \infty)$	1
2	$\{2, 3\}$	$(0, 3, 1, \infty)$	2
3	$\{3, 4\}$	$(0, 3, 1, 5)$	3
4	$\{4, 2\}$	$(0, 2, 1, 4)$	4
5	$\{2\}$	$(0, 2, 1, 4)$	2
	$\emptyset$	$(0, 2, 1, 4)$	

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Properties of the GSPA…

Proposition 2.2: Consider the generic shortest path algorithm.

(a) At the end of each iteration, the following conditions hold:

(i) If $d_j < \infty$, then d_j is the length of some path that starts at 1 and ends at j.

(ii) If $i \notin V$, then either $d_i = \infty$ or else

$$d_j \leq d_i + a_{ij}, \qquad \forall\ j \text{ such that } (i,j) \in \mathcal{A}.$$

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...Properties of the GSPA

(b) If the algorithm terminates, then upon termination, for all j with $d_j < \infty$, d_j is the shortest distance from 1 to j and

$$d_j = \begin{cases} \min_{(i,j)\in\mathcal{A}}\{d_i + a_{ij}\} & \text{if } j \neq 1, \\ 0 & \text{if } j = 1. \end{cases} \tag{2.4}$$

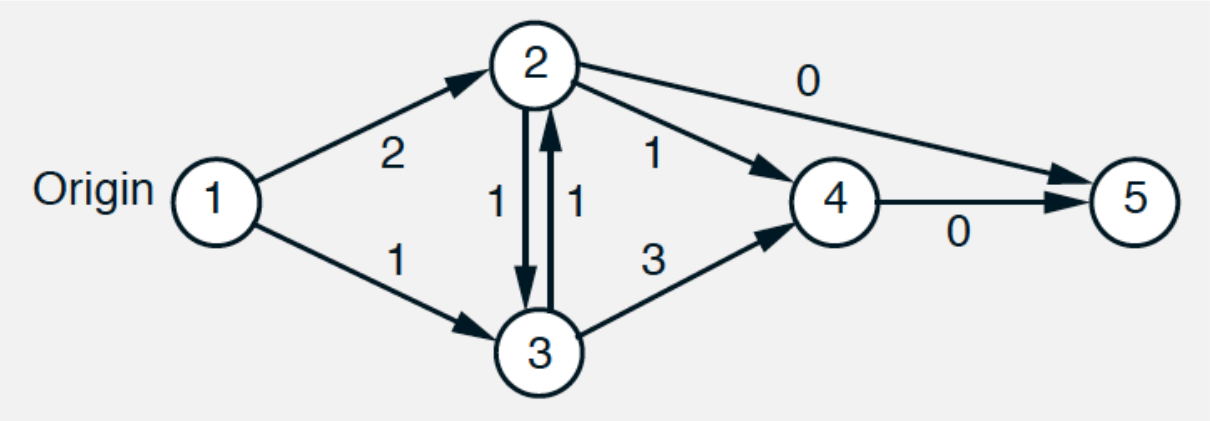
Furthermore, upon termination we have $d_j = \infty$ if and only if there is no path from 1 to j.

(c) If the algorithm does not terminate, then there exists some node j and a sequence of paths that start at 1, end at j, and have lengths that diverge to $-\infty$.

(d) The algorithm terminates if and only if there is no path that starts at 1 and contains a cycle with negative length.

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Example of Dijkstra Algo: Figure 2.5



Iteration #	Candidate List V	Node Labels	Node out of V
1	$\{1\}$	$(0, \infty, \infty, \infty, \infty)$	1
2	$\{2, 3\}$	$(0, 2, 1, \infty, \infty)$	3
3	$\{2, 4\}$	$(0, 2, 1, 4, \infty)$	2
4	$\{4, 5\}$	$(0, 2, 1, 3, 2)$	5
5	$\{4\}$	$(0, 2, 1, 3, 2)$	4
	$\emptyset$	$(0, 2, 1, 3, 2)$	

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Properties of the Dijkstra Algo...

Proposition 2.3: Assume that all arc lengths are nonnegative.

(a) For any iteration of the label setting method, the following hold for the set

$$W = \{i \mid d_i < \infty, i \notin V\}. \tag{2.9}$$

(i) No node belonging to W at the start of the iteration will enter the candidate list V during the iteration.

(ii) At the end of the iteration, we have $d_i \leq d_j$ for all $i \in W$ and $j \notin W$.

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...Properties of the Dijkstra Algo

(iii) For each node j, consider simple paths that start at 1, end at j, and have all their other nodes in W at the end of the iteration. Then the label d_j at the end of the iteration is equal to the length of the shortest of these paths ($d_j = \infty$ if no such path exists).

(b) The label setting method will terminate, and all nodes with a final label that is finite will be removed from the candidate list V exactly once in order of increasing shortest distance from node 1; that is, if the final labels of i and j are finite and satisfy $d_i < d_j$, then i will be removed before j.

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The Bellman-Ford property

Bellman-Ford Property

For each node i and integer $k \geq 1$, let

d_i^k = Shortest distance from 1 to i using paths that have k arcs or less,

where $d_i^k = \infty$ if there is no path from 1 to i with k arcs or less. Then the label d_i at the end of the kth cycle of iterations of the Bellman-Ford method is less or equal to d_i^k.